Math 121A   Midterm 1   October 2, 2023

Name: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points, 2 points each) Determine whether the following sequences or seires converge or not.

   (a) $\lim_{x\to\infty} x^n e^{-x}$

   (b) $\lim_{n\to\infty} (1/n)^{1/n}$

   (c) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

   (d) $\sum_n 1/(n \log n)$

   (e) $\sum_n e^n/\sqrt{n!},$

2. (10 points, 2 points each)

   (1) Use z and $\bar{z}$ to express $\mathrm{Re}(z), \mathrm{Im}(z), |z|^2$.

   (2) If $z = 2023 + 1002i$, then $|z/\bar{z}| =?$.

   (3) If $z = (1/2)e^{i\pi/3}$, then $1/\bar{z} =?$

   (4) Write down the solutions for $z^3 = e^{i\pi/2}$.

   (5) If $z = 1 + 2i$, is $\cos(z) = Re(e^{iz})$?

3. (10 points, 5 points each) Suppose

$$f(z) = z/(z+1)^2$$

   (1) Do the Taylor expansion of $f(z)$ at $z = 0$ and keep the first two non-zero terms.

   (2) Do the Laurent expansion of $f(z)$ at $z = -1$ and keep the first two non-zero terms.

4. (10 points) Compute

$$\oint_{|z|=2} \frac{1}{z(z+1)(z+4)(z+5)} dz$$

5. (10 points) Compute

$$\int_{-\infty}^{\infty} \frac{1}{1+ix^2} dx$$

2