Math 121A Final Dec 18, 2019

Name: ____

- The exam time is 3 hours, 8am - 11am.
- This is a closed-book exam. No notes, books, calculators, computers, or electronic aids are allowed.
- Please write neatly. Answers which are illegible for the reader cannot be given credit.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Compute the Maclaurin expansion of

$$f(x) = \frac{1}{(1-x)(1-2x)}.$$

   Then find its radius of convergence.

2. Compute the following integral

$$\oint_{|z|=1} \frac{e^{\sin z}}{z} dz$$

3. Compute the principal value of the following integral

$$P.V. \int_{-\infty}^{\infty} \frac{x \sin x}{9x^2 - \pi^2} dx$$

4. Compute the Fourier transformation $\hat{f}(p)$ for

$$f(x) = \frac{1}{1+x^2}.$$

where the Fourier transformation is defined as

$$\hat{f}(p) = (2\pi)^{-1} \int_{-\infty}^{\infty} f(x) e^{-ixp} dx.$$

5. Find the (exponential) Fourier series for the following periodic function $f(x)$ in

$$f(x) = \frac{1}{1 - 2e^{ix}}$$

That is, write $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$. You may (or may not) find the following formula useful

$$c_n = (2\pi)^{-1} \int_0^{2\pi} f(x) e^{-inx} dx.$$

6. Find the general solution to the following equation

$$y' + y\cos x = \sin 2x$$

7. Use Laplace transformation to solve differential equation

$$y'' + 4y' + 4y = 4, \quad x \in (0, \infty)$$

where $y(0) = 0, y'(0) = 1$. The following might be helpful

$$LT(y) = Y(p), \quad LT(y') = pY(p) - y(0), \quad LT(y'') = p^2Y(p) - py(0) - y'(0).$$

$$LT(1) = 1/p, \quad LT(e^{-at}) = 1/(p + a).$$

8. Find the Green's function for the boundary value problem on $[0, 1]$

$$y''(x) + y'(x) = f(x), \quad y(0) = y(1) = 0$$

9. Use Fermat's principle to find the path followed by a light ray if the index of refraction n is proportional to y^{-1}. That is, write down and solve the Euler equation for

$$I = \int y^{-1} ds$$

10. Let $f(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-1)^2/2}$ and $g(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-2)^2/2}$. Find the convolution $f \star g$. Recall that

$$(f \star g)(x) = \int_{\mathbb{R}} f(y) g(x-y) dy$$

You may find the following formula useful

$$\int_{\mathbb{R}} e^{-\frac{x^2}{2\sigma^2}} dx = \sqrt{2\pi}\sigma, \quad \sigma > 0.$$