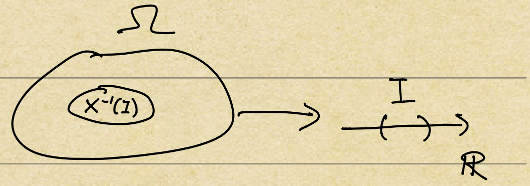
Random variable: (continuous R.V.)

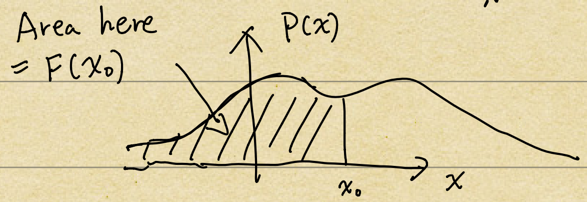


- $(\Omega, \mathbb{P})$. sample space + probability
- RV. $X: \Omega \to \mathbb{R}$. Induces a probability on $\mathbb{R}$.

$(\mathbb{R}, \mathbb{P}_X)$. $\mathbb{P}_X(I) = \mathbb{P}(X^{-1}(I))$
$I \subset \mathbb{R}$

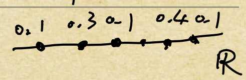
* probability density function (pdf):
  - $p(x) = P_X(x) = \lim\limits_{\varepsilon \to 0} \dfrac{\mathbb{P}_X(X \in [x, x+\varepsilon])}{\varepsilon}$
  - positive.
  - $\int_{-\infty}^{+\infty} p(x)\, dx = 1$
* Cumulative density function (CDF):

$$F(x) := \mathbb{P}_X(X < x) = \int_{-\infty}^{x} p(x)\, dx.$$



<u>Discrete R.V.</u> : the random variable takes value in a discrete set in $\mathbb{R}$. e.g. $\{1, 2, 3, 7, 10\} \subset \mathbb{R}$ finite set

or $\mathbb{Z} \subset \mathbb{R}$ is an example of infinite discrete set. point masses



- there is no <u>density function</u>. instead, we talk about $\mathbb{P}(X = x_i)$ where x_i is a possible value.

eg: flip coin 10 times, and count the # of heads.
that is a R.V. with possible values (a.k.a "range")
$\{0, 1, \cdots, 10\}$.

Numerical Characteristics:

- <u>Expectation value</u>. X : R.V.

$$\mathbb{E}(X) := \begin{cases} \int_{-\infty}^{+\infty} x \cdot p(x) \cdot dx \\ \sum\limits_{x \in \text{Range}} x \cdot \mathbb{P}(X = x) \end{cases}$$

mean.
" μ. μ_X.

- Variance : to measure fluctuation.

$$Var(X) := \mathbb{E}\left((X-\mu)^2 \right) \qquad \mu = \mathbb{E}(X).$$

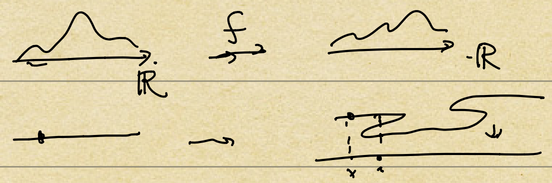
$$= \int_{-\infty}^{+\infty} (x-\mu)^2 \cdot p(x) \cdot dx .$$

notation: $\mathbb{E}(f(X))$

$$= \int_{-\infty}^{+\infty} f(x) \cdot p(x) \cdot dx$$

if f is a function, then $f(X)$ is another random variable.

$f : \mathbb{R} \to \mathbb{R}$.



- Moment : k-th moment of X

$$m_k(X) := \mathbb{E}(X^k).$$

$$m_1(X) = \mathbb{E}(X) = \mu.$$

$$\mathbb{E}((X-\mu)^2) = \mathbb{E}(X^2 - 2\mu X + \mu^2) = \mathbb{E}(X^2) - 2\mu \cdot \mathbb{E}(X) + \mu^2$$

$$= m_2 - 2 \cdot \mu^2 + \mu^2 = m_2 - \mu^2 = m_2 - m_1^2$$

- Moment Generating Function (MGF) : a function in an auxillary variable t,

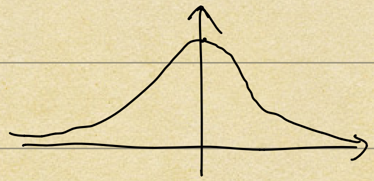
$$G(t) := \mathbb{E}(e^{tX}) = \mathbb{E}\left(1 + tX + \frac{t^2}{2} \cdot X^2 + \frac{t^3}{3!} X^3 + \cdots \right)$$

$$= 1 + m_1 \cdot t + \frac{t^2}{2} \cdot m_2 + \frac{t^3}{3!} \cdot m_3 + \cdots$$

Famous Continuous R.V. :

- Gaussian R.V.

standard normal : PDF $p(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$.



$\mu = 0$

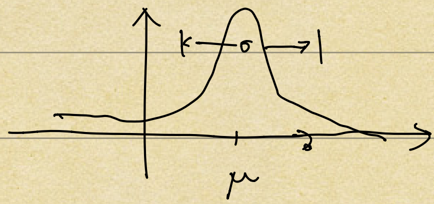
$\sigma^2 = var(X)$

$$= \int \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \cdot x^2 \cdot dx$$

use Γ function. $= 1.$

more generally, for other mean μ and variance σ^2.

$$P(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



- Γ - R.V. (param = α, β)

$$p(x) = C_{\alpha,\beta} \cdot x^{\alpha-1} \cdot e^{-\beta x}. \qquad x \in (0, \infty).$$

C constant, so that $\int p(x)\,dx = 1$.

- β - R.V. $\quad p(x) = C \cdot x^{p-1} \cdot (1-x)^{q-1} \qquad x \in [0,1]$

(p, q are param.)

- Uniform Distribution on an interval $[a, b]$.

$$P(x) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0. & \text{else} \end{cases}$$