Stochastic Process:

- Defn: it is a sequence of random variables.

  $\{X_t, t \in I\}$ I: index set. (for example $I = \mathbb{N}$, or $I = \mathbb{R}$)
  discrete time, continuous time.

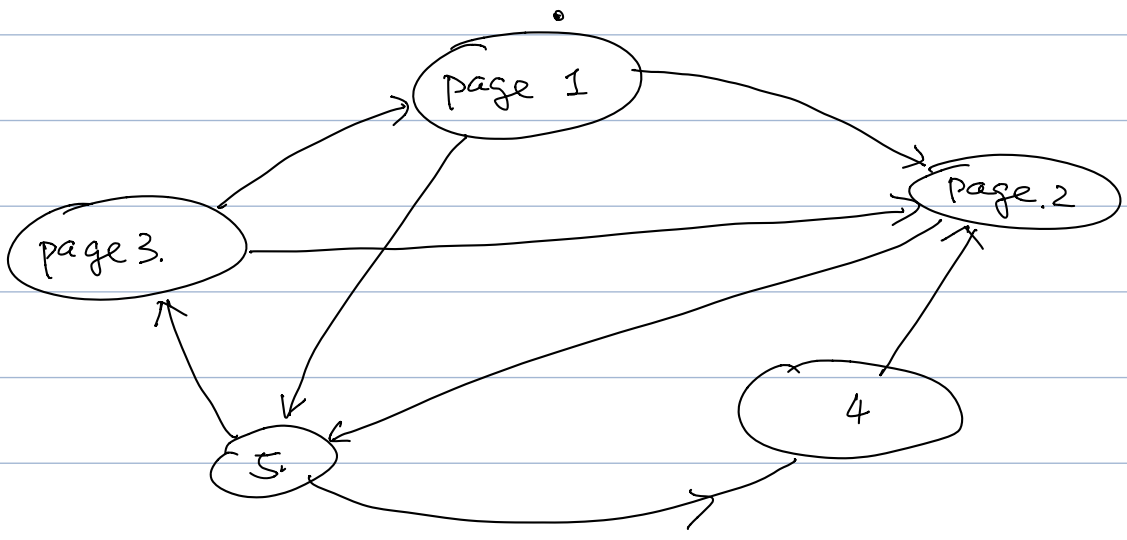
  all X_t takes value in the same space, "S" the state space.

  "random function: $X : I \to S$"

  (omitted causality in the definition).

- Ex 1: "Surf the Web" and "Page Rank".

  Suppose you are google, and you want rank the webpages on website based on its popularity / traffic. A simple model is based on "links"



- Create a graph (with nodes and arrows) with node = website, arrow = links.

- Assume a webuser browse the page, and randomly follows the link,

Question: what is the probability (percentage of traffic) of each page?

· ⇔ do a random walk on the graph, want to find the the average (stay time / total time spent) on each node.
(Markov Chain).

Ex 2: Spread of Infectious Disease model.

Assumption: ① Population of N people.

② $N = S$ (susceptible. healthy, not infected yet)
$+ I$ (infected.)
$+ R$ (recovered: will not be infected).

(simplifying assumption).

③ · after being infected for one time unit, one will recover.

· each susceptible person will contact every body during the time unit. so the probability of get infected from one contact is $\underline{\underline{z}}$.

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S_t, I_t, R_t. $t = 0, 1, 2, 3, \cdots$

· $N = S_t + \underline{I_t} + R_t$. so only need S_t. I_t

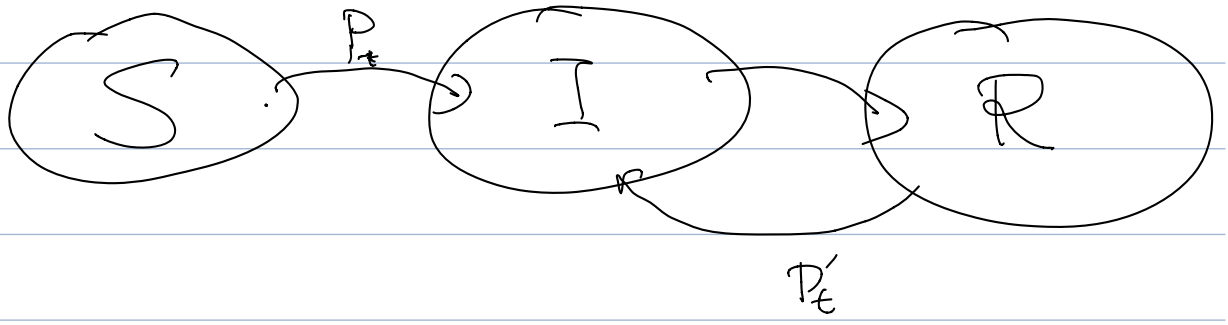
· I_{t+1} = the newly infected ones during time $t \sim t+1$,
= Binomial distribution. of sample size S_t.
with prop $\underline{\underline{p}}$.

p: probability that $\underline{\underline{a}}$ susceptible person get infected in time. $t \sim t+1$ | − (prop of no infection) = | − $(1-z)^{I_t}$

P.

I_{t+1} : is a random vaible, follows a distribution of Binom $(S_t, 1-(1-\gamma)^{I_t})$.

$S_{t+1} = S_t - I_{t+1}$.



- compartmental disease. (wikipage).

- Random walk / Brownian Motion / Gambler's Ruin.

- Gambler enters a casino with initial amount of money, k. He want to get final amount n.

The game is flipping coin. with prop of winning $= p$.

X_t = amount of money left at time t.

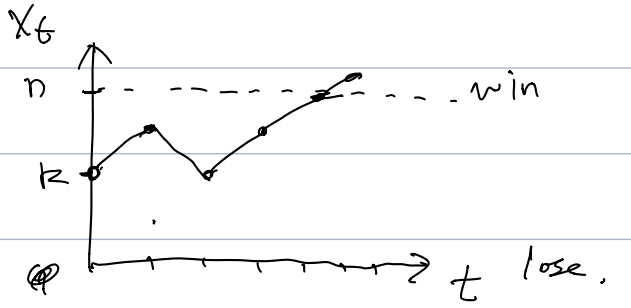
$X_0 = k$, $X_{t+1} = X_t + Y_{t+1}$

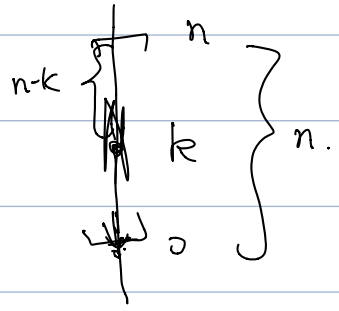
$$Y_{t+1} = \begin{cases} +1 & \text{prob } p \\ -1 & 1-p \end{cases}$$

Y_t. i.i.d. (independent. identical distribution) R.V.

Bernoulli(p).

Q: what is the probability the Gambler has no money when he leaves?





(assume $p = \frac{1}{2}$)

probability of ruin $= \frac{n-k}{n}$