No.
Counterexample:

$$\begin{aligned}
&\frac{1}{1}z^1 - \frac{1}{2}z^2 \\
&+ \frac{1}{3}z^3 + \frac{1}{4}z^4 - \frac{1}{5}z^5 - \frac{1}{6}z^6 \\
&+ \frac{1}{7}z^7 + \frac{1}{8}z^8 + \frac{1}{9}z^9 - \frac{1}{10}z^{10} - \frac{1}{11}z^{11} - \frac{1}{12}z^{12} \\
&+ \frac{1}{13}z^{13} + \frac{1}{14}z^{14} + \frac{1}{15}z^{15} + \frac{1}{16}z^{16} - \frac{1}{17}z^{17} - \frac{1}{18}z^{18} - \frac{1}{19}z^{19} - \frac{1}{20}z^{20} \\
&\cdots
\end{aligned}$$

We may also write this as

$$\sum_{1 \leq n}^{<\infty} s_n \frac{1}{n} z^n$$

where

$$s_n = \begin{cases} +1 & \text{if } k(k-1) \leq n < k^2 \text{ f.s. } k \in \mathbb{N} \\ -1 & \text{if } k^2 \leq n < k(k+1) \text{ f.s. } k \in \mathbb{N} \end{cases}$$

We may also write this as

$$\sum_{1 \leq k}^{<\infty} \left(\sum_{k(k-1) \leq n}^{<k^2} \frac{1}{n} z^n - \sum_{k^2 \leq n}^{<k(k+1)} \frac{1}{n} z^n \right)$$

This series diverges absolutely at 1.
I invite you to prove that it converges at every z with $|z| = 1$.