Problem 1

Compute

a) $(3 + 4i) \cdot (1 - 2i)$,

b) $e^{i2\pi/3} + e^{-i2\pi/3}$,

c) the argument of $\frac{(1+i)^3}{1-i}$.

Solution

a) $(3 + 4i) \cdot (1 - 2i) = 3 - 6i + 4i - 8i^2 = 11 - 2i$

b) $e^{i2\pi/3} + e^{-i2\pi/3} = 2\cos(2\pi/3) = -1$

c) Note $1 + i = \sqrt{2}e^{i\pi/4}$ and $1 - i = \sqrt{2}e^{-i\pi/4}$, so

$$\frac{(1+i)^3}{1-i} = 2e^{-i\pi}$$

so the argument is $-\pi$.

Problem 2

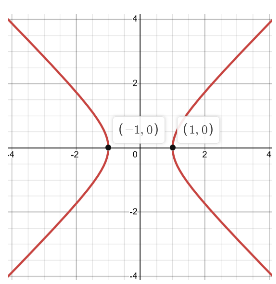
Sketch the region

$$\Re(z^2) = 1$$

and argue why the region sketched is correct.

Solution

Write $z = x + iy$. Then, $z^2 = (x + iy)(x + iy) = x^2 = 2ixy - y^2$. Hence, $\Re(z^2) = x^2 - y^2 = 1$. So, the region is a hyperbola.



Problem 3

Compute $(1 + i)^n + (1 - i)^n$ in terms of cosine. For what n is this quantity zero?

Hint: It may be useful to recall that any complex number $z \in \mathbb{C}$ can be expressed as $z = re^{i\theta}$ for suitable r, θ. You may also need to recall Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$.

Solution

Note that $1+i = \sqrt{2}e^{i\pi/4}$ and $1-i = \sqrt{2}e^{-i\pi/4}$. Hence,

$$(1+i)^n + (1-i)^n = \sqrt{2}^n (e^{in\pi/4} + e^{-in\pi/4}) = \sqrt{2}^n \cdot 2\cos(n\pi/4).$$

This is zero when $n\frac{\pi}{4} = k\pi + \frac{\pi}{2}$ for any $k \in \mathbb{Z}$; i.e. $n = 4k+2$.