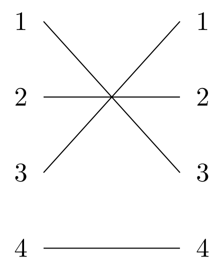
Problem 1

Compute the sign of the permutation $\sigma = (3214)$ by first computing the length. Compute its inverse.

Solution

Let $\ell(\sigma)$ be the length of the permutation. Then the sign is $\varepsilon(\sigma) = (-1)^{\ell(\sigma)}$. We can find the length by counting the number of crossing in the graph:



Since there are 3 crossings, $\ell(\sigma) = 3$. Hence, $\varepsilon(\sigma) = -1$. Note that σ keeps 2 and 4 fixed and just swaps 1 and 3. Hence, $\sigma^2 = \text{id}$ and so $\sigma^{-1} = \sigma$; i.e. σ is its own inverse.

Problem 2

Compute the determinant of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

Solution

$$\begin{aligned} \det(A) &= 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} - 0 \cdot \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} + 2 \cdot \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} \\ &= -1 - 8 = -9 \end{aligned}$$

Problem 3

Compute the determinant of the following matrix

$$B = \begin{bmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 1 & -\lambda \end{bmatrix}$$

Solution

$$\begin{aligned}
\det(B) &= (1-\lambda)\cdot\begin{vmatrix}2-\lambda & 1\\ 1 & -\lambda\end{vmatrix} - 0\cdot\begin{vmatrix}0 & 1\\ 2 & -\lambda\end{vmatrix} + 2\cdot\begin{vmatrix}0 & 2-\lambda\\ 2 & 1\end{vmatrix}\\
&= (1-\lambda)((2-\lambda)(-\lambda)-1) + 2(-(2-\lambda)2)\\
&= -\lambda^3 + 3\lambda^2 + 3\lambda - 9
\end{aligned}$$