Problem 1

Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that the area of the parallelogram defined by $T(0,1)$ and $T(1,0)$ is $\frac{1}{2}$.

Bonus: (5pts) Using your solution to the first part or otherwise, find a family of linear transformations with this property.

Solution

Problem 2

Compute the determinant of the following $(n+m) \times (n+m)$ matrix

$$\begin{bmatrix} 0 & A \\ B & C \end{bmatrix},$$

where A is an $n \times n$ matrix and B is $m \times m$.

Solution

Problem 3

Compute the determinant of the following matrix

$$A = \begin{vmatrix} \lambda & -1 & 0 & \cdots & 0 \\ 0 & \lambda & -1 & \cdots & 0 \\ \vdots & \dots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda & -1 \\ a_n & a_{n-1} & \dots & a_2 & \lambda + a_1 \end{vmatrix}$$

Hint: Use induction.

Solution