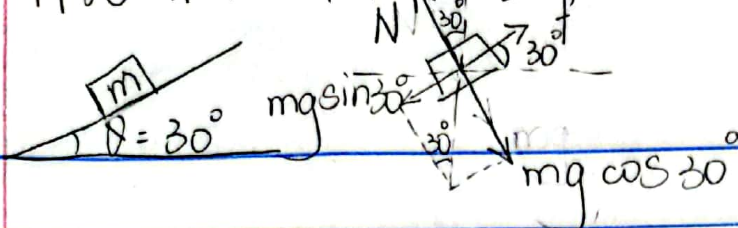
HW #1 . p.7 in [LA] Q# 1-12

1.



Assume $g = -10\ m/s^2$

f, N are magnitudes of $\vec{F_f}, \vec{F_N}$

$\vec{F_G} = m\vec{g} = (0, -10m)$

Since the mass m rests on the inclined plane, the net force on it is 0, which means that

$$\begin{cases} \vec{F_f} = -m\vec{g}\sin 30^\circ = f(\cos 30^\circ, \sin 30^\circ) = f(\frac{\sqrt{3}}{2}, \frac{1}{2}) \\ \vec{F_N} = -m\vec{g}\cos 30^\circ = N(-\sin 30^\circ, \cos 30^\circ) = N(-\frac{1}{2}, \frac{\sqrt{3}}{2}) \\ \vec{F_f} + \vec{F_N} = -m\vec{g} = (0, 10m) \end{cases}$$

$\hookrightarrow f(\frac{\sqrt{3}}{2}, \frac{1}{2}) + N(-\frac{1}{2}, \frac{\sqrt{3}}{2}) = (0, 10m)$

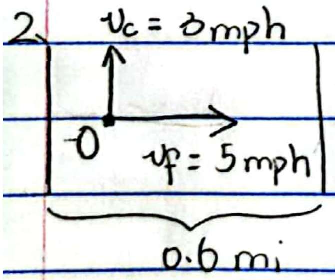
$$\begin{cases} \frac{\sqrt{3}}{2}f - \frac{1}{2}N = 0 \\ \frac{1}{2}f + \frac{\sqrt{3}}{2}N = 10m \end{cases} \Rightarrow \begin{matrix} \sqrt{3}f = N \\ f + \sqrt{3}N = 20m \end{matrix} \Rightarrow f + \sqrt{3}(\sqrt{3}f) = 20m$$

$\therefore 4f = 20m \qquad N = 5\sqrt{3}\,m$

$f = 5m$

Above all, $\vec{F_f} = (\frac{5\sqrt{3}}{2}m, \frac{5}{2}m)$, $\vec{F_N} = (-\frac{5\sqrt{3}}{2}m, \frac{15}{2}m)$

2.

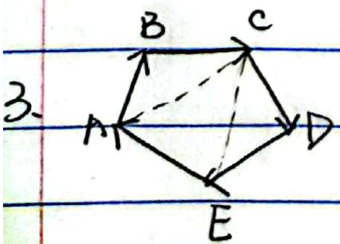


the total length of a round trip = 1.2 mi

$\because t = \frac{s}{v}$

$\therefore t = \frac{1.2\,mi}{5\,mph} = 0.24\,h$

3.



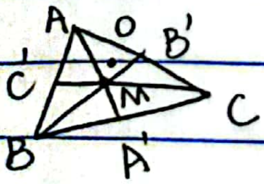
$\vec{AB} + \vec{BC} = \vec{AC}$, $\vec{CD} + \vec{DE} = \vec{CE} \rightarrow \vec{AC} + \vec{CE} = \vec{AE} = -\vec{EA}$

$\therefore \vec{AB} + \vec{BC} + \cdots + \vec{DE} + \vec{EA} = -\vec{EA} + \vec{EA} = 0$

Now assume that we have n+1 lines

$\therefore \vec{A_1A_2} + \vec{A_2A_3} + \cdots + \vec{A_{n-1}A_n} + \vec{A_nA_{n+1}} + \vec{A_{n+1}A_1} = 0$

4.



$\vec{OM} = \vec{OA} + \vec{AM} = \vec{OB} + \vec{BM} = \vec{OC} + \vec{CM}$, $\vec{MA} + \vec{MB} + \vec{MC} = 0$

$\therefore 3\vec{OM} = \vec{OA} + \vec{OB} + \vec{OC} + (\vec{AM} + \vec{BM} + \vec{CM}) \rightarrow \vec{OM} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$