HW #3

[ODE] Ex. 1.3.4

[LA] p22 q# 44, 45, 47, 48, 49, 51, 54, 57, 58

Ex. 1.3.4

a) $x_1 + 2x_2 = 3$

$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \to R_{\pi/4} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}x_1 + \frac{\sqrt{2}}{2}x_2 \\ \frac{\sqrt{2}}{2}x_1 - \frac{\sqrt{2}}{2}x_2 \end{bmatrix}$

$\frac{\sqrt{2}}{2}x_1 + \frac{\sqrt{2}}{2}x_2 + 2\left(\frac{\sqrt{2}}{2}x_1 - \frac{\sqrt{2}}{2}x_2\right) = 3$

$\sqrt{2}x_1 + \sqrt{2}x_2 + 2\sqrt{2}x_1 - 2\sqrt{2}x_2 = 6$

$3\sqrt{2}x_1 - \sqrt{2}x_2 = 6$

$6x_1 - 2x_2 = 6\sqrt{2} \to 3x_1 - x_2 = 3\sqrt{2}$

b) basis $f_1 = T_\theta e_1$, $f_2 = T_\theta e_2$

In the basis e_1, e_2

$R_0 = \begin{bmatrix} \cos 0 & \sin 0 \\ \sin 0 & -\cos 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

In the basis f_1, f_2

$R_0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

c) $(u_1+u_2)(v_1-v_2) = u_1v_1 + u_2v_1 - u_1v_2 - u_2v_2$ bilinear

$(u_1+v_1)(u_2+v_2) = u_1u_2 + v_1u_2 + u_1v_2 + v_1v_2$ both

$(u_1+u_2)(v_1+v_2) = u_1v_1 + u_2v_1 + u_1v_2 + u_2v_2$ both

d) Prove $(A^t)^t = A$?

$\langle \vec{v}, A^t\vec{u} \rangle = \langle A^t\vec{u}, \vec{v} \rangle = \langle \vec{u}, (A^t)^t\vec{v} \rangle$

$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \cdot \begin{bmatrix} a_{11}u_1 + a_{21}u_2 \\ a_{12}u_1 + a_{22}u_2 \end{bmatrix} = a_{11}v_1u_2 + a_{21}v_1u_2 + a_{12}v_2u_1 + a_{22}v_2u_2$