Problem 1. A mass m rests on an inclined plane making 30°with the horizontal plane. Find the forces of friction and reaction acting on the mass.

I never took physics... and I'm new to linear algebra... and Latex... and Overleaf... so take this answer (and how I got there) with a grain of salt, but, based off the answer in the textbook for this problem, I surprisingly achieved the textbook's solution.

Please see under *"Useful Links"* in my notebook for a link to a website that helped me a lot for this problem (as well as the triangle diagram ("Figure 2") that I also used for this problem.)

Defining Variables:

F_g = force of gravity (calculated by mg, where m = mass and g = gravity constant)
F_N = Normal force (always perpendicular to the surface the object is on) (equal to reaction force, I believe, in this case... again, I never took physics, so I apologize if this is wrong...)
F_f = force of friction
$F_{\parallel}$ = component of F_g parallel to slope (magnitude is the same as F_f but opposite direction) (see "Figure 2" in the link posted for this problem)
$F_{\perp}$ = component of F_g perpendicular to the slope (magnitude is the same as F_N but opposite direction) (see "Figure 2")

To find $F_{\parallel}$: We will use the formula $(F_g)(\cos 30°)$, in which F_g is, as listed above, equal to mg, where m is mass and g is the gravity constant (both "unknown" at this time). To find cos 30°, either use your calculator or use the 30-60-90 triangle rule, in which the side opposite the 30°angle is 1/2, the side opposite the 60°angle is $\sqrt{3}/2$, and the side opposite the 90°angle is 1.

The resulting equation is: 1/2(mg) = mg/2 = -mg/2 (because the direction of $F_{\parallel}$ is going to the left (see "Figure 2")).

As a result, F_f (or the force of friction) is mg/2, as its direction is the opposite of F_f.

We will follow the same process to find the force of reaction by using $F_{\perp}$, but instead using the formula $(F_g)(\sin 30°)$, resulting in (mg)($\sqrt{3}/2$) = (mg)($\sqrt{3}$)/2 = -(mg)($\sqrt{3}$)/2 (as the direction of $F_{\perp}$ is going downwards).

Thus, F_N or the Normal Force (or the reaction force, in this case) is: (mg)($\sqrt{3}$)/2.

Problem 2. A ferry capable of making 5 *mph*, shuttles across a river of width 0.6 *mi* with a strong current of 3 *mph*. How long does each round trip take?

This problem is actually really simple – don't overthink it lol. We will assume the river is flowing horizontally (so the river's current is flowing vertically). We know the value of the vertical vector is 3 (as the current is flowing at 3 mph). We also know the value of the diagonal vector is 5 (as the boat is moving at 5 mph). Therefore, to find the rate of a trip from one side to the other (or the magnitude of the horizontal vector), we use the Pythagorean Theorem ($3^2 + x^2 = 5^2$), giving us the value of the horizontal vector: 4.

Thus, the time the boat moves at 4 mph to complete a trip of 0.6 miles. There are 1.2 miles in a round trip (0.6 + 0.6). We will use the formula *distance = rate × time.*

1.2 miles = 4 mph × x hours

Thus, we get 0.3 hours (or 18 minutes). Therefore, it takes the boat 18 minutes to make the round trip.

Problem 3. Prove that for every closed broken line ABC ... DE,

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \mathbf{0}$

Closed broken lines are broken lines who endpoints coincide... basically, they form a polygon (see link for Problem 2 under "Useful Links" for more info).

We are trying to prove that, when adding all the vectors, the vectors equal zero. During addition, vectors only equal zero when the vectors have the same magnitude but opposite direction. For example: $\overrightarrow{AB}$ + $\overrightarrow{BA}$ = $\overrightarrow{AA}$0.

So, we are going to want to end up with vectors that have the same vector but opposite directions, so that when we add them up, they we will equal zero. Since $\overrightarrow{EA}$ is the last vector listed, let's try to get to an equation that ends up with only $\overrightarrow{AE}$ + $\overrightarrow{AE}$ = 0.

Let's work with this equation: $\overrightarrow{AB}$ + $\overrightarrow{BC}$ + $\overrightarrow{CD}$ + $\overrightarrow{DE}$ + $\overrightarrow{EA}$. If you remember from class, adding the two vectors $\overrightarrow{AB}$ + $\overrightarrow{BC}$ = $\overrightarrow{AC}$. So, we can simplify $\overrightarrow{AB}$ + $\overrightarrow{BC}$ to simply $\overrightarrow{AC}$.

So far, the equation now looks like: $\overrightarrow{AC}$ + $\overrightarrow{CD}$ + $\overrightarrow{DE}$ + $\overrightarrow{EA}$.

Okay, now let's add $\overrightarrow{AC}$ + $\overrightarrow{CD}$. That equals $\overrightarrow{AD}$. Now, our equation is: $\overrightarrow{AD}$ + $\overrightarrow{DE}$ + $\overrightarrow{EA}$ = 0.

Finally, let's add $\overrightarrow{AD}$ + $\overrightarrow{DE}$. That gives you what we were looking for: $\overrightarrow{AE}$. Now, our equation is: $\overrightarrow{AE}$ + $\overrightarrow{EA}$ = 0. Like I said before, adding $\overrightarrow{AE}$ + $\overrightarrow{EA}$ = $\overrightarrow{AA}$ = 0.

Problem 4. Three medians of a triangle ABC intersect at one point M called the barycenter of the triangle. Let O be any point on the plane. Prove that:

$\overrightarrow{OM} = 1/3(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$.

If you want to use Problem 5 to solve Problem 4, please see Bryan's or Ryan's solution for this lol... I tried to solve Problem 4 without using it. Also, I knew nothing about barycenters before this... so I had to do a lot of research too... and **used a lot of Math StackExchange for this question, the link to the one which I used is in the "Useful Links" for this problem**. So essentially, for this question (except for the beginning portion), it's me trying to reason out what has already been solved/proven... so, again, I'd like to reiterate for this question, except for the beginning portion of the question, the rest is solved and credited to user Math Lover on StackExchange.

Note: A barycenter is the mean of all the vertices. Basically, (A + B + C) / 3 = M, *where* M *is the centroid or where the three medians of the vertices meet (or the barycenter). To prove that the barycenter splits the medians of the vertices into ratios of 2:1, please watch the khan academy video I linked under "Useful Links" for this problem.*

Draw a triangle, labelling the vertices A, B, and C. Draw the medians (a line beginning from one of the vertices that intersects the opposite side at its midpoint). Label the midpoint of B and C: A_1. For me, I drew O on the same line as A and C for simplicity's sake. (Refer to the StackExchange for this picture)

O is an arbitrary point on the same plane. So, $\overrightarrow{OM}$ = $\overrightarrow{OA}$ + $\overrightarrow{AM}$, $\overrightarrow{OM}$ = $\overrightarrow{OB}$ + $\overrightarrow{BM}$, and $\overrightarrow{OM}$ = $\overrightarrow{OC}$ + $\overrightarrow{CM}$.

We know that, according to the definition of a barycenter, that $\overrightarrow{AM}$ = 2/3 × $\overrightarrow{AA_1}$. Since $\overrightarrow{OM}$ = $\overrightarrow{OA}$ + $\overrightarrow{AM}$, we can substitute 2/3 × $\overrightarrow{AA_1}$ for $\overrightarrow{AM}$, resulting in the equation: $\overrightarrow{OM}$ = $\overrightarrow{OA}$ + 2/3 × $\overrightarrow{AA_1}$. To get everything over 3 (to make it easier to get 1/3 at the end), we simplify (or complicate the equation to): (3$\overrightarrow{OA}$ + 2$\overrightarrow{AA_1}$) / 3. And since $\overrightarrow{OA} + \overrightarrow{AA_1} = \overrightarrow{OA_1}$, we can substitute that in, leaving 1 $\overrightarrow{OA}$ in the equation: ($\overrightarrow{OA}$ + 2$\overrightarrow{OA_1}$) / 3.

Now, we can see if we can get $\overrightarrow{OA_1}$ in terms of $\overrightarrow{OB}$ and $\overrightarrow{OC}$. $\overrightarrow{OA_1}$ = $\overrightarrow{OB}$ + $\overrightarrow{BA_1}$. So, now we can include $\overrightarrow{OB}$ in the equation. To include $\overrightarrow{OC}$ in the equation, let's look at $\overrightarrow{BA_1}$, where, according to the fact that point A_1 is the midpoint of $\overrightarrow{BC}$, $\overrightarrow{BA_1}$ is equal to $\overrightarrow{A_1C}$, which is then equal to $\overrightarrow{OC}$ - $\overrightarrow{OA_1}$. So now you have, as a substitute of $\overrightarrow{OA_1} = \overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA_1}$, which then gets you: $\vec{2}OA_1 = \overrightarrow{OB} + \overrightarrow{OC}$.

This gives you the equation: ($\overrightarrow{OA}$ + $\overrightarrow{OB}$ + $\overrightarrow{OC}$) / 3 = $\overrightarrow{OM}$ or 1/3($\overrightarrow{OA}$ + $\overrightarrow{OB}$ + $\overrightarrow{OC}$) = $\overrightarrow{OM}$

Problem 5 Prove that $\overrightarrow{MA}$ + $\overrightarrow{MB}$ + $\overrightarrow{MC}$ = $\mathbf{0}$ if and only if M is the barycenter of the triangle ABC.

I'm going to be using what we learned from **Problem 7** for this bc it was tiring/painful to do **Problem 4** without **Problem 5**.

According to the definition of a barycenter (see **Problem 4.** for more info), $\overrightarrow{MA}$ is $2/3(\overrightarrow{AA_1})$, $\overrightarrow{MB}$ is $2/3(\overrightarrow{BB_1})$, and $\overrightarrow{MC}$ is $2/3(\overrightarrow{CC_1})$. Therefore, if you substitute these values in, you get:

$2/3(\overrightarrow{AA_1}) + 2/3(\overrightarrow{BB_1}) + 2/3(\overrightarrow{CC_1}) = 0$, which is then equal to $2/3(\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1}) = 0$.

Using **Question 7**, you can substitute: $\overrightarrow{AA_1} = (1/2(\overrightarrow{AB} + \overrightarrow{AC}))$, $\overrightarrow{BB_1} = (1/2(\overrightarrow{BA} + \overrightarrow{BC}))$, and $\overrightarrow{CC_1} = (1/2(\overrightarrow{AC} + \overrightarrow{BC}))$.

This gives you the equation: $(2/3)(((1/2(\overrightarrow{AB} + \overrightarrow{AC})) + (1/2(\overrightarrow{BA} + \overrightarrow{BC})) + (1/2(\overrightarrow{AC} + \overrightarrow{BC})))$. Simplified, you get: $(2/3)(1/2)(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{AC} + \overrightarrow{BC})$.

Similar to what we proved in **Problem 3**, everything in the parentheses gives you zero, leaving you with the equation $(2/3)(1/2)(0)$, which equals 0.

Problem 6. Along three circles lying in the same plane, vertices of a triangle are moving clockwise with the equal constant angular velocities. Find how the barycenter of the triangle is moving.

This problem is actually really simple. As someone who never took physics though, I had to look up angular velocity, which essentially is the vector measure of how fast an object rotates around another point (basically the rate). Every point on the object has the same angular velocity... which essentially answers the question. See the link in "*Useful Links*" for more info.

So, the barycenter of the triangle is moving with the same angular velocity vector in a circle compared to the vertices of the triangle formed by the three other circles on the same plane.

Problem 7. Prove that if AA' is a median in a triangle ABC, then:

$\overrightarrow{AA'} = 1/2(\overrightarrow{AB'} + \overrightarrow{AC})$

This is actually pretty simple. We know that $\overrightarrow{AA'} = \overrightarrow{AB} + \overrightarrow{BA'}$ and that $\overrightarrow{AA'} = \overrightarrow{AC} + \overrightarrow{CA'}$. And... if we end up adding $\overrightarrow{BA'} + \overrightarrow{CA'}$, we will get 0 because they are the same magnitude with the opposite direction. So, how do we get the 1/2 into the equation?

$\overrightarrow{AA'} = 1/2(\overrightarrow{AA'} + \overrightarrow{AA'})$. Substitute in for $\overrightarrow{AA'}$ and we get: $\overrightarrow{AA'} = 1/2(\overrightarrow{AB} + \overrightarrow{BA'} + \overrightarrow{AC} + \overrightarrow{CA'}$.

And, like I said above, because $\overrightarrow{BA'} + \overrightarrow{CA'} = 0$, our final equation is: $1/2(\overrightarrow{AB} + \overrightarrow{AC}) = \overrightarrow{AA'}$.