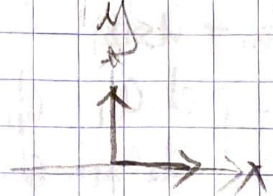
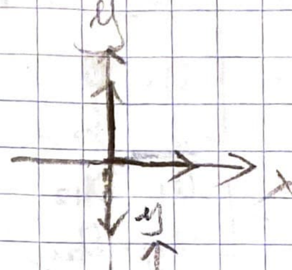
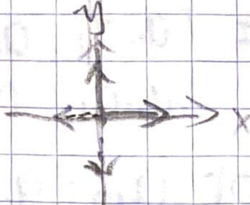
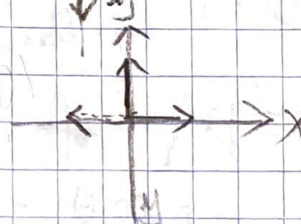
HW2:

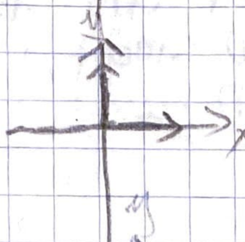
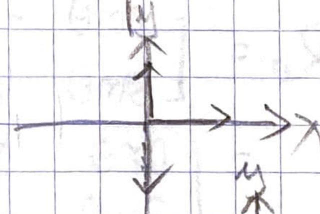
Exercise 1.3.1:

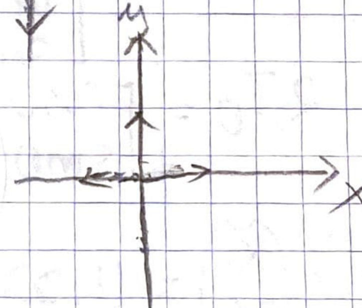
(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ 

$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$  $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ 

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ 

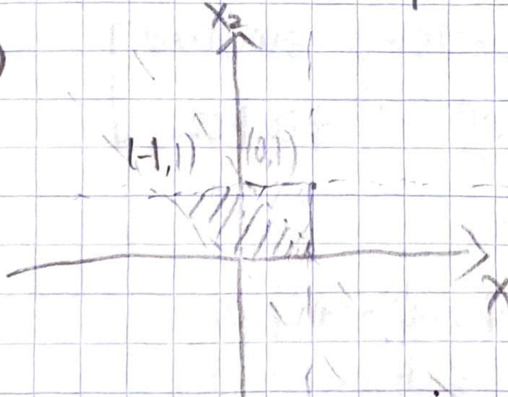
$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ 

(c) $\tilde{x}_1 = x_1 - x_2 \quad \tilde{x}_2 = x_1 + x_2$

$\tilde{x}_1 \tilde{x}_2 = (x_1 - x_2)(x_1 + x_2) = x_1^2 - x_2^2$

$$\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

(d)



$$\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$\langle Tx, Tx \rangle = \langle x, x \rangle \quad \langle Ty, Ty \rangle = \langle y, y \rangle$

(g) $\langle x+y, x+y \rangle = \langle Tx+Ty, Tx+Ty \rangle$

$\langle x, x \rangle + \langle y, y \rangle + 2\langle x, y \rangle = \langle Tx, Tx \rangle + \langle Ty, Ty \rangle + 2\langle Tx, Ty \rangle$

$\langle x, y \rangle = \langle Tx, Ty \rangle$

Exercises 1.3.2

(a) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$

(b) $DA = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} d_1 a_{11} & d_1 a_{12} \\ d_2 a_{21} & d_2 a_{22} \end{bmatrix}$

$AD = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} = \begin{bmatrix} a_{11}d_1 & a_{12}d_2 \\ a_{21}d_1 & a_{22}d_2 \end{bmatrix}$

$AD = DA$ iff $d_1 = d_2$ or $a_{12} = a_{21} = 0$ (i.e. A is diagonal)

(c) $T_\theta R_0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos 0 & \sin 0 \\ \sin 0 & -\cos 0 \end{bmatrix}$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} = R_\theta$

$R_0 T_\theta = \begin{bmatrix} \cos 0 & \sin 0 \\ \sin 0 & -\cos 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ \sin(-\theta) & -\cos(-\theta) \end{bmatrix}$

$= R_{-\theta}$

(d) $\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = (1,2) \begin{bmatrix} \cos(-\frac{\pi}{3}) & -\sin(-\frac{\pi}{3}) \\ \sin(-\frac{\pi}{3}) & \cos(-\frac{\pi}{3}) \end{bmatrix} \begin{bmatrix} \cos\pi & \sin\pi \\ \sin\pi & -\cos\pi \end{bmatrix}$

$= (1,2) \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \left(\sqrt{3} - \frac{1}{2}, \frac{\sqrt{3}}{2} + 1\right)$

(e) $T_\phi T_\psi = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$

$= \begin{bmatrix} \cos\phi\cos\psi - \sin\phi\sin\psi & -\cos\phi\sin\psi - \sin\phi\cos\psi \\ \sin\phi\cos\psi + \cos\phi\sin\psi & -\sin\phi\sin\psi + \cos\phi\cos\psi \end{bmatrix}$

(f) $R_\phi R_\psi = \begin{bmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}$

$= \begin{bmatrix} \cos\phi\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\psi - \sin\phi\cos\psi \\ \sin\phi\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\psi + \cos\phi\cos\psi \end{bmatrix}$

$= \begin{bmatrix} \cos(\phi-\psi) & -\sin(\phi-\psi) \\ \sin(\phi-\psi) & \cos(\phi-\psi) \end{bmatrix}$

$= T_{\phi-\psi}$