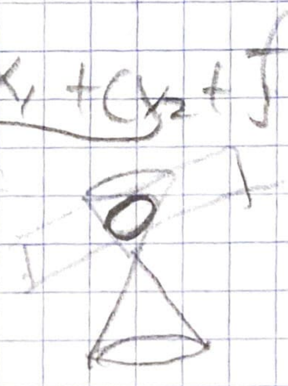
Discussion:

Conics:

Solns to quadratic expressions

$$\underbrace{ax_1^2+2bx_1x_2+cx_2^2}_{QF}\underbrace{+dx_1+cx_2}_{LF}+f=0$$



$$x_1^2+x_2^2=x_3^2$$

$$x_3=ax_1+bx_2$$

Principal Axis Theorem: Any QF in 2 variables has two perpendicular axes

Orthogonal Diagonalization Theorem: Any QF can be espressed as $Ax_1^2+Bx_2^2$ in a suitable coordinate system.

ODT $\Rightarrow$ PAT

Corollary: Any QF can be expressed in the "normal form"

$$x_1^2+x_2^2,\ x_1^2-x_2^2,\ -x_1^2-x_2^2,\ x_1^2,\ x_2^2,\ 0$$

Discussion:

Determinants

det: $M_n(K) \to K$

1) $\det(I) = 1$

2) If $B = A$ except $row_i(B) = c \cdot row_i(A)$

$\det(B) = c \cdot \det(A)$

3) $A = B = C$ except in row i we have

$row_i(A) = row_i(B) + row_i(C)$

$\det(A) = \det(B) + \det(C)$

4) If B arises from A by swapping 2 rows, then $\det(B) = -\det(A)$

Thm: a) The determint of a matrix with a row of all zeroes is 0

b) " with two identical rows is 0

c) " with one row a multiple of another row is 0

d) " If a multiple of one row of a matrix is added to another, the determinant remains the same

Thm: Uniqueness of the determinant

Proof: Let A be a matrix with at least one non zero entry in each row. By multi linearity we know

$\det(A) = \det(A_1) + \cdots + \det(A_n)$

where A_j is the matrix identical to A except in the ith row, it is all zeroes except for the jth element