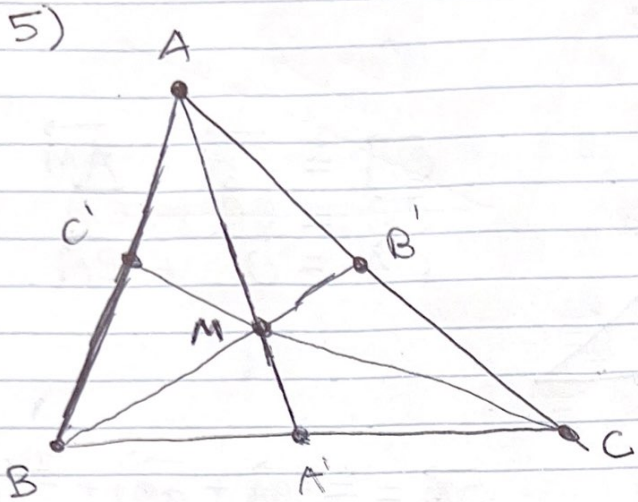
Homework #1: Ch. 1 Vectors

5)



$\overrightarrow{AA'} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

$\overrightarrow{BB'} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC})$

$\overrightarrow{CC'} = \frac{1}{2}(\overrightarrow{CB} + \overrightarrow{CA})$

$\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC}) + \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC}) + \frac{1}{2}(\overrightarrow{CB} + \overrightarrow{CA})$

$\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BA} + \overrightarrow{AC} + \overrightarrow{CA} + \overrightarrow{BC} + \overrightarrow{CB})$

$\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = \frac{1}{2}(\vec{0} + \vec{0} + \vec{0})$

$\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = \vec{0}$

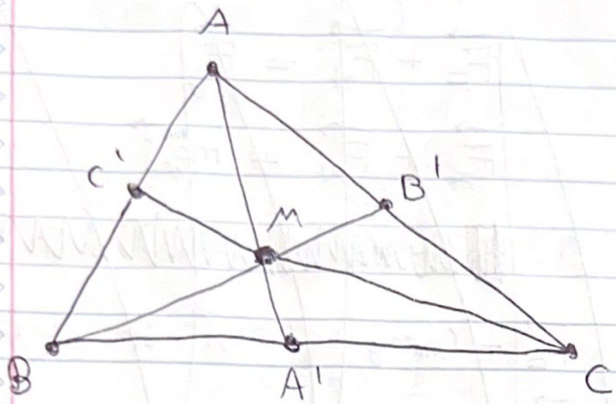
$\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = \frac{2}{3}(\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'})$ by Centroid Ratio Theorem

$= \frac{2}{3}(\vec{0}) = \vec{0}$

Hence $\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = \vec{0}$ if and only if M is the barycenter of ΔABC, because otherwise the Centroid Ratio Theorem would not hold.

Homework #1: Ch. 1 Vectors (cont.)

4)



Suppose O is a point lying on the same plane as $\triangle ABC$. Then

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$
$$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$$
$$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$$

Thus,

$$\overrightarrow{OM} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{AM}) + \frac{1}{3}(\overrightarrow{OB} + \overrightarrow{BM}) + \frac{1}{3}(\overrightarrow{OC} + \overrightarrow{CM})$$

$$\overrightarrow{OM} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{AM} + \overrightarrow{BM} + \overrightarrow{CM})$$

$$\overrightarrow{OM} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \vec{0})$$ see #5

$$\overrightarrow{OM} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$$

7) Refer to the same figure used in #4

$$\overrightarrow{AA'} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$$

$$2\overrightarrow{AA'} = \overrightarrow{AB} + \overrightarrow{AC}$$

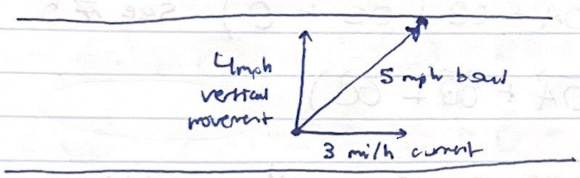
$$2\overrightarrow{AA'} - \overrightarrow{AB} - \overrightarrow{AC} = \vec{0}$$

$$\overrightarrow{AA'} - \overrightarrow{AB} + \overrightarrow{AA'} - \overrightarrow{AC} = \vec{0}$$

$$\overrightarrow{A'B} + \overrightarrow{A'C} = \vec{0}$$

$$\vec{0} = \vec{0}$$

2)



$$\frac{1.2 \text{ mi}}{4 \frac{\text{mi}}{\text{h}}} = 0.3 \text{ h} = 18 \text{ mins}$$