

4) Set $\vec{OA} = (a_1, a_2)$

$\vec{OB} = (b_1, b_2)$

$\vec{OC} = (c_1, c_2)$

$$\frac{1}{2}(\vec{AB} + \vec{AC}) = \left\langle \frac{b_1+c_1}{2} - a_1, \frac{b_2+c_2}{2} - a_2 \right\rangle$$

$$= \vec{OA}' - \vec{OA} = \vec{AA}'$$

similarly $\vec{AA}' = \frac{1}{2}(\vec{AB} + \vec{AC})$

$\vec{BB}' = \frac{1}{2}(\vec{BA} + \vec{BC})$

$\vec{CC}' = \frac{1}{2}(\vec{CA} + \vec{CB})$

$$\therefore \vec{AA}' + \vec{BB}' + \vec{CC}' = 0$$

$$\frac{2}{3}(\vec{AA}' + \vec{BB}' + \vec{CC}') = 0$$

$$\vec{MA}' + \vec{MB}' + \vec{MC}' = 0$$

$$\vec{OA} = \vec{OM} + \vec{MA}$$

$$\vec{OB} = \vec{OM} + \vec{MB}$$

$$\vec{OC} = \vec{OM} + \vec{MC}$$

$$\vec{OA} + \vec{OB} + \vec{OC} = 3\vec{OM}$$

$$\vec{OM} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$$

10) $\vec{DE} = \vec{A'A} = \frac{1}{2}(\vec{BA} + \vec{CA})$

$\vec{DF} = \vec{B'B} = \frac{1}{2}(\vec{BA} + \vec{BC})$

$$\frac{1}{2}(\vec{DE} + \vec{DF}) = \frac{1}{4}\vec{BA} + \frac{1}{4}\vec{CA} + \frac{1}{4}\vec{BA} + \frac{1}{4}\vec{BC}$$

$$\vec{DD}' = \frac{3}{4}\vec{BA}$$

similarly $\vec{EE}' = \frac{3}{4}\vec{AC}$

$\vec{FF}' = \frac{3}{4}\vec{BC}$

\therefore The medians are also parallel to the sides.

11) Set $\vec{OA} = (a_1, a_2)$ $\vec{OB} = (b_1, b_2)$ $\vec{OC} = (c_1, c_2)$ $\vec{OD} = (d_1, d_2)$

$\vec{OE} = (e_1, e_2)$

AB midpoint M $\left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}\right)$

CD midpoint N $\left(\frac{c_1+d_1}{2}, \frac{c_2+d_2}{2}\right)$

MN midpoint X $\left(\frac{a_1+b_1+c_1+d_1}{4}, \frac{a_2+b_2+c_2+d_2}{4}\right)$

\therefore slope of MN = $\frac{h_1+c_1+d_1+e_1}{4} \cdot \frac{h_2+c_2+d_2+e_2}{4}$ ✓

> similarly, we can get $(\frac{a_1 - a_2}{4}, \frac{a_2 - a_1}{4})$ as 1

$$\times \left(\frac{e_1 - a_1}{4}, \frac{e_2 - a_2}{4} \right) = \frac{1}{4} \vec{AE}$$

12) if X lies on AB

then $\vec{XB} = \lambda \vec{AB}$ and $0 \leq \lambda \leq 1$

$$\vec{OB} - \vec{OX} = \lambda (\vec{OB} - \vec{OA})$$

$$\vec{OX} = \lambda \vec{OA} + \vec{OB} - \lambda \vec{OB}$$

$$= \lambda \vec{OA} + (1 - \lambda) \vec{OB}$$