Peng Zhou   Office hour : Tue 12:30 - 13:30
Evans 753

How diff. Eq. relates to linear algebra

e.g. $f''(x) = f(x)$

solution is linear combination of e^x and e^{-x}

Linear Algebra → formal structures of Math

↳ Linear operations : $\times$, $+$

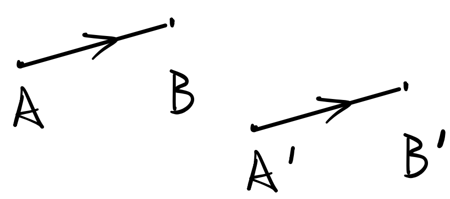
Vectors → magnitude & direction

column matrix

an element for vector space

vectors on a plane

→ directed segments $\vec{AB}$

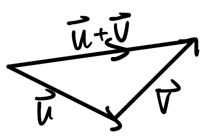


→ translation : move the end points

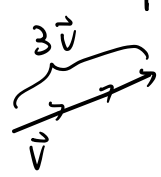
vector is an equivalence class of directed segment

2 directed segments differ by translation ⇒ same vector

Vectors addition



scalar multiplication



linear combinations

$\vec{v}$. $\vec{w}$ are vectors

$a . b \in \mathbb{R}$

$a\vec{v} + b\vec{w}$ is well defined.

- Inner product / dot product $\vec{v} \cdot \vec{u}$

given 2 vectors $\vec{v}$ and $\vec{u}$

we define $\langle \vec{v}, \vec{u} \rangle$

$\langle \vec{v}, \vec{u} \rangle = |\vec{v}||\vec{u}| \cos\theta$

→ linear operation

↳ structure of scalar multiplication & addition

e.g. $\langle \vec{v}_1 + \vec{v}_2 , \vec{u} \rangle$

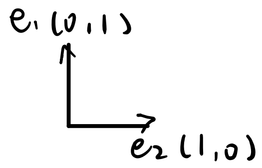
$= \langle \vec{v}_1 , \vec{u} \rangle + \langle \vec{v}_2 , \vec{u} \rangle$

$\langle 3\vec{v} , \vec{w} \rangle = 3 \langle \vec{v}, \vec{w} \rangle$

$\langle \vec{u}, \vec{u} \rangle = |\vec{u}|^2$ $\langle \vec{u}_1 + \vec{u}_2 , \vec{v}_1 + \vec{v}_2 \rangle$

$$\cos\theta = \frac{\langle \vec{u}, \vec{v} \rangle}{|\vec{u}| \cdot |\vec{v}|} \qquad = \langle \vec{u}_1, \vec{v}_1 \rangle + \langle \vec{u}_2, \vec{v}_2 \rangle + \langle \vec{u}_1, \vec{v}_2 \rangle + \langle \vec{u}_2, \vec{v}_1 \rangle$$

Basis vectors



Any other vectors can be written in the linear combination of e_1 and e_2

$(e_1, e_2) \Rightarrow$ ortho normal spaces

$e_1 \perp e_2$ $\quad |e_1| = |e_2| = 0$

Claim : $\vec{V} = \langle \vec{v}, e_1 \rangle e_1 + \langle \vec{v}, e_2 \rangle e_2$

Suppose $V = a_1 e_1 + a_2 e_2$

apply $\langle \cdot , e_1 \rangle$ to both sides

$$\begin{aligned} \langle \vec{V}, e_1 \rangle &= \langle a_1 e_1 + a_2 e_2 , e_1 \rangle \\ &= \langle a_1 e_1, e_1 \rangle + \langle a_2 e_2, e_1 \rangle \\ &= a_1 |e_1|^2 + a_2 \cdot 0 \\ &= a_1 \end{aligned}$$

Skewed basis on $\mathbb{R}^2$

if (E_1, E_2) in $\mathbb{R}^2$ satisfies $\forall\ V = a_1 E_1 + a_2 E_2$

then (E_1, E_2) is a basis

Coordinates : given a basis (E_1, E_2)

we say vector $\vec{V}$ has coordinate (a_1, a_2)

if $\vec{V} = a_1 \vec{E}_1 + a_2 \vec{E}_2$