[LA] p22: 44, 45, 47, 48, 49, 51, 54, 57, 58

44. complex numbers can be real, real & imaginary, neither

$5 + 0i \qquad 0 + 0i \qquad 5 + 5i$

45 $(1+i)/(3-2i) = \frac{(1+i)(3+2i)}{(3-2i)(3+2i)} = (3+5i-2)/(9+4) = \frac{1}{13}(1+5i)$

a) $(1+i)/(3-2i) = \frac{1}{13}(1+5i)$

b) $(\cos \pi/3 + i \sin \pi/3)^{-1} = \frac{1}{\cos \pi/3 + i \sin \pi/3} = \frac{1}{\frac{1}{2} + \frac{\sqrt{3}}{2} i} = \frac{\frac{1}{2} - \sqrt{3}/2\, i}{\frac{1}{4} + 3/4} = \frac{\frac{1}{2} - \sqrt{3}/2\, i}{1} = \boxed{\frac{1}{2} - \sqrt{3}/2\, i}$

47. $z = re^{i\theta} \quad \bar{z} = re^{i-\theta} \quad z^{-1} = (re^{i\theta})^{-1} = \frac{1}{r} e^{-i\theta}$

$\bar{z}/z^{-1} = \frac{re^{i-\theta}}{\frac{1}{r} e^{i-\theta}} = \boxed{r^2}$

48. $|z-1| = |z+1| = 2 \qquad |z-1|^2 = |z+1|^2 = 4$ $\cancel{|(x+yi)+1|^2 = |(x+1)+iy|^2 = (x+1)^2 + y^2}$

$\cancel{|(x+yi)-1|^2 = |(x-1)+iy|^2 = (x-1)^2 + y^2}$

$\cancel{1 + x^2 + 2x + y^2 = x^2 - 2x + 1 + y^2}$

$\cancel{4x = 4}$

$|(x+yi)-1|^2 = 4 \qquad |(x+yi)+1|^2 = 4$

$|(x-1) + yi|^2 = 4 \qquad |(x+1) + yi|^2 = 4$

$x^2 - 2x + 1 + y^2 = 4 \qquad x^2 + 2x + 1 + y^2 = 4$

$x^2 - 2x - 3 + y^2 = x^2 + 2x - 3 + y^2$

$4x = 0$

$\boxed{x = 0;\ y = \pm\sqrt{3}\, i}$

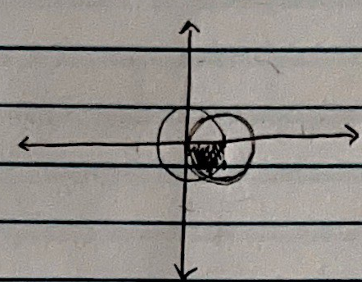
49. $|z-1| \le 1$

$\cancel{|(x-1)+iy|^2 \le 1}$

$\cancel{(x-1)^2+y^2 \le 1}$

$\cancel{x^2-2x+1+y^2 \le 1}$

$\cancel{x^2-2x+y^2 \le 0}$



$|z| \le 1$ is the circle around $O$ w/ rad. 1

$|z-1| \le 1$ is the circle around $(1,0)$ w/ rad 1

$Re(iz) \Rightarrow iz = i(x+iy) = ix - y = -y + ix = -y$

$-y \le 0$

51. $\left(\frac{\sqrt{3}+i}{2}\right)^{100} = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{100} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{100} = e^{(i\,\pi/6)^{100}} = e^{i\,50\pi/3} = \cos^{50\pi/3} + i\sin^{50\pi/3}$

$= \cos\left(16\pi + \frac{2\pi}{3}\right) + i\sin\left(16\pi + 2\pi/3\right) = \boxed{-\frac{1}{2} + i\sqrt{3}/2}$

$\frac{S}{T} \Big| \frac{A}{C}$