Pricing Mechanisms for Economic Dispatch: A Game-Theoretic Perspective

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Abstract

The economic dispatch problem is to determine a socially optimal allocation of supply and demand subject to transmission constraints. With strategic generators possessing market power, the locational marginal pricing mechanism may not induce the economic dispatch. We investigate the market outcomes from a game-theoretic perspective, employing piecewise constant offer curves as in practice, in contrast to affine offer curves as in the literature. We present counterexamples to show that a Nash equilibrium may not exist, or that the price of anarchy can be arbitrarily large. On the other hand, we provide sufficient conditions under which there exist efficient Nash equilibria. We then study the LMP mechanism with Cournot offers. As an alternative, we propose the marginal contribution pricing mechanism that guarantees efficient outcomes.

Keywords: Cournot competition, economic dispatch, electricity market, game theory, locational marginal pricing, supply function equilibrium

1. Introduction

Electric power is traded in a wholesale electricity market that involves various entities: the generators who generate and sell power, the distributors who buy power and sell to consumers, and the independent system operator (ISO) who coordinates the operation of the market and the power system. The generators and the distributors submit economic signals for supply and demand to the ISO, who then determines an optimal dispatch that maximizes the social welfare while satisfying the physical and operational constraints. This problem is referred to as the economic dispatch problem.

While the economic dispatch is computed, the price paid (or charged) to each generator (or distributor) is determined accordingly. Locational marginal pricing (LMP) is a widely used mechanism for pricing electricity in the wholesale electricity market [1, 2]. When the market is competitive, the LMP mechanism is efficient, i.e., inducing the economic dispatch.

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However, those market participants are strategic agents who potentially possess market power. They may not have incentives to reveal their true characteristics, or may even provide misleading information for their own interests. For example, Enron's energy traders found ways to manipulate the congestion prices that led to the California electricity crisis of 2000–01 [3]. While market power can be measured using market power indices [4], those indices have their own limitations and may not adequately characterize the situation of market manipulation.

In this work, we adopt a game-theoretic approach to study the LMP mechanism through investigating the equilibrium outcomes. We focus on the strategic bidding of the supply side, assume general cost functions, and take the transmission constraints into account. Our model differs from the supply function equilibrium (SFE) models [5, 6], in that we employ piecewise constant offer curves as in practice, in contrast to affine offer curves as in the SFE literature. Moreover, we explore the existence and the efficiency of Nash equilibria, while most of the existing work focuses on computing SFEs [7, 8, 9, 10]. Our main results are twofold. On the one hand, we show that a Nash equilibrium may not exist, or that the price of anarchy can be arbitrarily large. On the other hand, we provide sufficient conditions under which there exist efficient Nash equilibria.

We then study the LMP mechanism with Cournot offers, in which generators submit quantity offers instead of price/quantity offer curves. Due to the tractability, Cournot models have gained popularity in electricity market analysis [11, 12, 13]. We establish the existence of Nash equilibria under certain mild conditions.

Furthermore, there are other mechanisms as alternatives to the LMP mechanism. For example, the pay-as-bid pricing mechanism is compared with the LMP mechanism in [14]. We propose the marginal contribution pricing (MCP) mechanism which always induces efficient Nash equilibria, using piecewise constant offer curves,

This paper is organized as follows. In Section 2, we introduce the economic dispatch problem and the LMP mechanism. We then formulate the economic dispatch game in Section 3. We provide counterexamples in Section 4 to illustrate the phenomena of market manipulation, and sufficient conditions in Section 5 which guarantee market efficiency. In Section 6, we study the LMP mechanism with Cournot offers. In Section 7, we propose the MCP mechanism as an alternative to the LMP mechanism. Finally, we present a case study in Section 8 to illustrate the equilibrium outcomes under various pricing mechanisms and offer formats. Section 9 concludes the paper.

2. Model

2.1. Economic Dispatch

We consider a connected power network which consists of I buses indexed by $i = 1, \ldots, I$, and N generators indexed by $n = 1, \ldots, N$. There can be zero, one, or more generators located at each bus i, the set of which is denoted by

N_i. For each generator n, the cost of supplying x_n is $c_n(x_n)$. The cost function $c_n : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed to be strictly increasing, convex and piecewise differentiable.

The competitive demand at each bus i is modeled by an inverse demand function $p^i(y_i)$, which maps the quantity y_i to the maximum price that the consumers are willing to pay. The inverse demand function $p^i : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed to be continuous and decreasing. It is sometimes convenient to work with the valuation function

$$v_i(y_i) = \int_0^{y_i} p^i(z)\mathrm{d}z,$$

which is increasing, concave and continuously differentiable. In this paper, we will also consider inelastic demand, in which case all the y_i's are fixed.

For analytical and computational simplicity, we adopt a DC power flow model as a common practice [15]. In the DC flow model, a branch i-j is characterized by B_{ij}, the negative of its susceptance, which satisfies $B_{ij} = B_{ji} \geq 0$. Let θ_i be the voltage phase angle at bus i. Then the active power flow over branch i-j is given by

$$f_{ij} = B_{ij}(\theta_i - \theta_j).$$

The bus power balance equation for bus i is the following:

$$\sum_{n \in N_i} x_n - y_i = \sum_j f_{ij}.$$

Let C_{ij} be the flow limit of branch i-j, which satisfies $C_{ij} = C_{ji} \geq 0$. The branch power flow constraint for branch i-j is the following:

$$f_{ij} \leq C_{ij}.$$

Let $x = (x_1, \ldots, x_N)$, $y = (y_1, \ldots, y_I)$, and $\theta = (\theta_1, \ldots, \theta_I)$. The economic dispatch problem is to determine an optimal allocation of supply and demand that maximizes the social welfare while satisfying the transmission constraints. Formally, it is modeled as a convex programming problem:

$$\begin{aligned}
\underset{x,y,\theta}{\text{maximize}} \quad & \sum_i v_i(y_i) - \sum_n c_n(x_n) && \text{(1a)}\\
\text{subject to} \quad & \sum_{n \in N_i} x_n - y_i = \sum_j B_{ij}(\theta_i - \theta_j),\ \forall i, && \text{(1b)}\\
& B_{ij}(\theta_i - \theta_j) \leq C_{ij},\ \forall (i,j), && \text{(1c)}\\
& x_n \geq 0,\ \forall n, && \text{(1d)}\\
& y_i \geq 0,\ \forall i. && \text{(1e)}
\end{aligned}$$

The primal optimal solution to (1) is called the economic dispatch, denoted by $(x^{**}, y^{**}, \theta^{**})$ throughout the paper.

Since $\sum_i(\sum_{n\in N_i} x_n - y_i) = \sum_i \sum_j B_{ij}(\theta_i - \theta_j) = 0$, the system of linear equations (1b) over i is underdetermined with respect to θ. In fact, only the phase angle differences matter. Thus, for computational purposes, we choose bus 1 as the slack bus by setting $\theta_1 = 0$.

The economic dispatch problem is based on the given on/off states of the generators, in comparison with the unit commitment problem. In the proposed model, the N generators have been started up, so that the fixed cost of each generator does not affect the dispatch. Thus, we further assume $c_n(0) = 0$ for all n without loss of generality. For a more detailed DC power flow model (e.g., where line losses are taken into account), the reader can refer to [16].

2.2. Locational Marginal Pricing

Associate the dual variables λ_i with (1b) and μ_{ij} with (1c). Let $\lambda = (\lambda_1, \ldots, \lambda_I)$ and $\mu = [\mu_{ij}]_{I\times I}$. It is easily seen that the economic dispatch problem (1) is always feasible, and that the (refined) Slater's condition is automatically satisfied. Therefore, strong duality holds, and the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality [17]:

$$(c_n'(x_n) - \lambda_i)x_n = 0,\ \forall n \in N_i, \tag{2a}$$

$$c_n'(x_n) - \lambda_i \geq 0,\ \forall n \in N_i, \tag{2b}$$

$$(v_i'(y_i) - \lambda_i)y_i = 0,\ \forall i, \tag{2c}$$

$$v_i'(y_i) - \lambda_i \leq 0,\ \forall i, \tag{2d}$$

$$\sum_j B_{ij}(\lambda_i - \lambda_j + \mu_{ij} - \mu_{ji}) = 0,\ \forall i, \tag{2e}$$

$$\sum_{n\in N_i} x_n - y_i - \sum_j B_{ij}(\theta_i - \theta_j) = 0,\ \forall i, \tag{2f}$$

$$\mu_{ij}(B_{ij}(\theta_i - \theta_j) - C_{ij}) = 0,\ \forall (i,j), \tag{2g}$$

$$B_{ij}(\theta_i - \theta_j) - C_{ij} \leq 0,\ \forall (i,j), \tag{2h}$$

$$x_n \geq 0,\ \forall n, \tag{2i}$$

$$y_i \geq 0,\ \forall i, \tag{2j}$$

$$\mu_{ij} \geq 0,\ \forall (i,j). \tag{2k}$$

Note that when c_n is not differentiable at x_n, $c_n'(x_n)$ should be interpreted as a subderivative, which always exists given the assumptions of the cost function.

Throughout the paper, denote by (λ^{**}, μ^{**}) the dual optimal solution to (1). Then $(x^{**}, y^{**}, \theta^{**}, \lambda^{**}, \mu^{**})$ satisfies the KKT conditions (2). The locational marginal price (LMP) at each bus i is given by λ_i^{**}, which is the cost of serving an additional increment of load at that bus. Thus, the payoff of generator n located at bus i is given by

$$\pi_n = \lambda_i^{**} x_n^{**} - c_n(x_n^{**}).$$

The LMP mechanism is efficient in a competitive environment, i.e., when generators are price takers. If the price at each bus i is fixed as λ_i^{**}, and each generator determines its supply to maximizes its own payoff, then the resulting allocation is the economic dispatch. This can be seen from the KKT conditions, which state that the marginal cost of a generator with positive generation is exactly the LMP at that bus.

3. Game-Theoretic Formulation

To solve (1) for the economic dispatch, the ISO needs to know the cost functions. However, strategic generators may not have incentives to reveal their true cost functions. They are often not even able to do so, since the offer is typically a low-dimensional signal that may not fully parametrize the cost function. This motivates the formulation of the economic dispatch game. We first specify the strategy space, or the offer format.

3.1. Offer Format

An offer curve $p_n(x_n)$ maps the quantity x_n to the minimum price that generator n is willing to accept. It is supposed to approximate the marginal cost function $c_n'(x_n)$, but not necessarily. The offer curve $p_n(x_n)$ corresponds to a reported cost

$$\hat{c}_n(x_n) = \int_0^{x_n} p_n(z)\mathrm{d}z.$$

The offer format refers to the parametrization of the offer curve, or equivalently, the parametrization of the reported cost.

In the SFE literature, the offer curve often takes an affine form:

$$p_n(x_n) = a_n + b_n x_n,\ a_n > 0,\ b_n > 0,$$

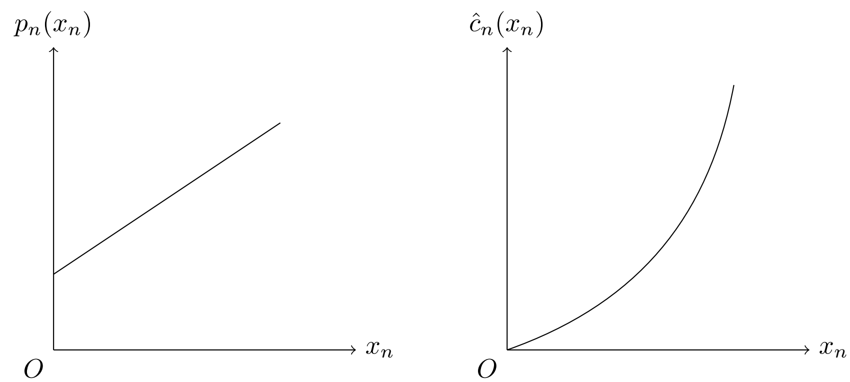
which corresponds to a quadratic reported cost

$$\hat{c}_n(x_n) = a_n x_n + (b_n/2)x_n^2.$$

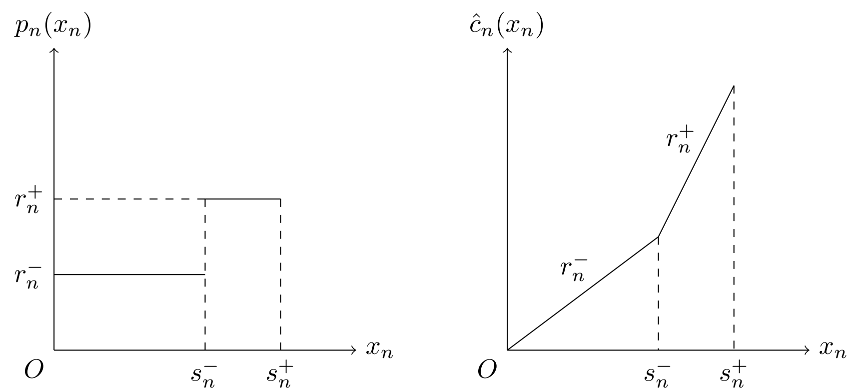
In practice, however, most ISOs adopt piecewise constant (or step) offer curves. Such an offer curve corresponds to a piecewise linear reported cost. For example, the California ISO asks for 10 price/quantity pairs [18], which correspond to a 10-segment piecewise linear reported cost.

In this paper, we employ the realistic offer format. For simplicity, we consider a two-segment piecewise linear reported cost parametrized by a four-dimensional signal $(r_n^-, s_n^-, r_n^+, s_n^+)$, where $0 \le r_n^- \le r_n^+, 0 \le s_n^- \le s_n^+$:

$$\hat{c}_n(x_n) = \begin{cases} r_n^- x_n, & x_n \in [0, s_n^-], \\ r_n^- s_n^- + r_n^+(x_n - s_n^-), & x_n \in (s_n^-, s_n^+]. \end{cases}$$



(a) Affine offer curve and quadratic reported cost.



(b) Piecewise constant offer curve and piecewise linear reported cost.

Figure 1: Comparison of offer formats used in the SFE literature and in this paper.

Fig. 1 illustrates the offer format used in the SFE literature and that used in this paper. We note that quadratic costs provide smooth dispatch, revenue and profit curves that facilitate calculus-based analysis, while piecewise linear costs do not produce continuously differentiable dispatch, revenue and profit curves, requiring different analysis techniques [19]. This distinguishes our work from the SFE literature.

3.2. Economic Dispatch Game

Let $\hat{c} = (\hat{c}_1, \ldots, \hat{c}_N)$ be the offer profile, i.e., the collection of reported cost. In reality, the ISO solves the following problem:

$$\begin{aligned}
\underset{x,y,\theta}{\text{maximize}} \quad & \sum_i v_i(y_i) - \sum_n \hat{c}_n(x_n) && \text{(3a)}\\
\text{subject to} \quad & \sum_{n \in N_i} x_n - y_i = \sum_j B_{ij}(\theta_i - \theta_j),\ \forall i, && \text{(3b)}\\
& B_{ij}(\theta_i - \theta_j) \leq C_{ij},\ \forall (i,j), && \text{(3c)}\\
& x_n \geq 0,\ \forall n, && \text{(3d)}\\
& y_i \geq 0,\ \forall i. && \text{(3e)}
\end{aligned}$$

Compared with (1), the true costs are replaced by the reported costs in (3). Let (x^*, y^*, θ^*) be the primal optimal solution to (3). The LMP at each bus i is given by λ_i^*, the optimal dual variable associated with the bus power balance equation (3b). Thus, the payoff of generator n located at bus i is given by

$$\pi_n = \lambda_i^* x_n^* - c_n(x_n^*).$$

This completes the specification of the economic dispatch game.

We adopt the pure strategy Nash equilibrium as the solution concept. An offer profile is a Nash equilibrium if no generator can get a higher payoff by a unilateral deviation. A Nash equilibrium is efficient if it induces the economic dispatch $(x^{**}, y^{**}, \theta^{**})$.

4. Market Manipulation

We have formulated an economic dispatch game to study the LMP mechanism with the realistic offer format. In this section, we illustrate the phenomena of market manipulation. Specifically, we present counterexamples to show that a Nash equilibrium may not exist, and that even when a Nash equilibrium exists, the price of anarchy can be arbitrarily large.

4.1. Nonexistence of Nash Equilibria

The following example is adapted from [7], where SFEs are computed with affine offer curves. Our focus, however, is on the existence of Nash equilibria with piecewise linear offer curves.

Example 1. *Consider the network as shown in Fig. 2. There are two buses, with generator* 1 *at bus* 1 *and generator* 2 *at bus* 2*. The cost functions are* $c_1(x_1) = 0.01x_1^2 + 10x_1$ *and* $c_2(x_2) = 0.01x_2^2 + 10x_2$*. There is no demand at bus* 1*. The inverse demand function at bus* 2 *is* $p(y) = 30 - 0.08y$*. The branch flow limit is* $C = 100$*. We prove by contradiction that a Nash equilibrium does not exist in this example.*

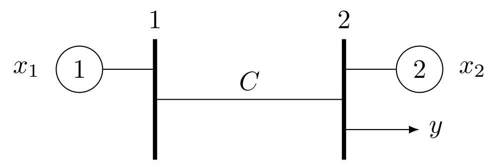


Figure 2: Example of a two-bus network in which a Nash equilibrium does not exist.

Suppose that a Nash equilibrium $(\hat{c}_1^*, \hat{c}_2^*)$ *exists, which induces the outcome* $(x_1^*, x_2^*, \lambda_1^*, \lambda_2^*)$*. Since there is no demand at bus* 1*,* $\lambda_1^* \leq \lambda_2^*$*. If* $\lambda_1^* < \lambda_2^*$*, it is easily seen that generator* 1 *has an incentive to deviate. Thus,* $\lambda_1^* = \lambda_2^*$*.*

Since $c_1'(C) = 12 < p(2C) = 14$*, the branch must be congested; otherwise, it can be shown that at least one of the generators has an incentive to deviate. Thus,* $x_1^* = 100, \lambda_1^* = \lambda_2^* \geq 12$*. Then* (x_2^*, λ_2^*) *must solve the following problem:*

$$\begin{array}{lll} \underset{x_2,\lambda_2}{\text{maximize}} & & \lambda_2 x_2 - c_2(x_2) \\ \text{subject to} & & \lambda_2 = p(x_1^* + x_2), \\ & & x_2, \lambda_2 \geq 0, \end{array}$$

which gives $x_2^* = 66.67, \lambda_2^* = 16.67$*. Since a portion of* x_1^* *is supplied at a reported marginal cost* $\lambda_1^* = \lambda_2^* = 16.67 > c_2'(x_2^*) = 11.33$*, it can be shown that generator* 2 *has an incentive to deviate: by decreasing* λ_2^* *by an arbitrarily small number* $\epsilon > 0$ *and increasing* x_2^**, generator* 2 *can get a higher payoff. Therefore, there does not exist a Nash equilibrium.*

Remark 1. *In this example, we illustrate a situation of market manipulation. The result is closely related to the concept of market power. Indeed, the nonexistence of Nash equilibria is due the exercise of market power of generator* 2*. However, not all market power indices are relevant. For example, the nodal must-run share (NMRS) [20] is undefined here, since the demand is elastic.*

4.2. Bad Price of Anarchy

There are also situations in which Nash equilibria exist but some of them are undesirable in terms of efficiency. The price of anarchy is a metric that measures how the efficiency degrades due to the strategic behavior, compared with the socially optimal outcome. Consider the case of inelastic demand, in which y_i is fixed for all i. Let (x^*, θ^*) be any equilibrium dispatch. The price of anarchy is defined as the ratio between the cost of the worst equilibrium and the socially optimal cost:

$$\text{PoA} = \frac{\max \sum_n c_n(x_n^*)}{\sum_n c_n(x_n^{**})},$$

where the maximum is taken over the set of equilibrium dispatch. The following example shows that the price of anarchy can be arbitrarily large.

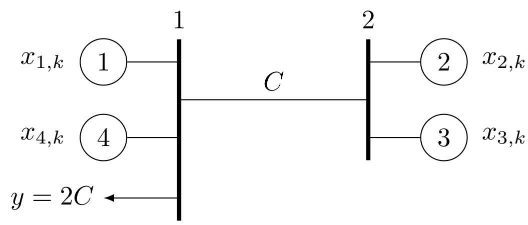


Figure 3: Example of a two-bus network in which the price of anarchy can be arbitrarily large.

Example 2. *Consider the network as shown in Fig. 3. The network has two buses, with generator* 1 *and* 4 *at bus* 1*, and generator* 2 *and* 3 *at bus* 2*. The branch flow limit is* C*. The inelastic demand at bus* 1 *is* $y = 2C$*. There is no demand at bus* 2*.*

Consider a sequence of games $\{\Gamma_k, k \geq 1\}$*. In game* Γ_k*, the cost functions are given by*

$$\begin{aligned} c_{1,k}(x_{1,k}) &= x_{1,k}, \\ c_{2,k}(x_{2,k}) &= kx_{2,k}, \\ c_{3,k}(x_{3,k}) &= kx_{3,k}, \\ c_{4,k}(x_{4,k}) &= 2kx_{4,k}. \end{aligned}$$

The economic dispatch is

$$(x_{1,k}^{**}, x_{2,k}^{**}, x_{3,k}^{**}, x_{4,k}^{**}) = (2C, 0, 0, 0),$$

with a social cost $2C$*. One Nash equilibrium is*

$$\begin{aligned} \hat{c}_{1,k}^*(x_{1,k}) &= 2kx_{1,k},\ \ x_{1,k} \leq 2C, \\ \hat{c}_{2,k}^*(x_{2,k}) &= kx_{2,k},\ \ x_{2,k} \leq 2C, \\ \hat{c}_{3,k}^*(x_{3,k}) &= kx_{3,k},\ \ x_{3,k} \leq 2C, \\ \hat{c}_{4,k}^*(x_{4,k}) &= 2kx_{4,k},\ \ x_{4,k} \leq 2C, \end{aligned}$$

which induces the outcome

$$(x_{1,k}^*, x_{2,k}^*, x_{3,k}^*, x_{4,k}^*) = (C, C, 0, 0),\ \ (\lambda_{1,k}^*, \lambda_{2,k}^*) = (2k, k),$$

with a social cost $C + kC$*. For example, generator* 1 *has no incentive to report* $\hat{c}_{1,k}(x_{1,k}) = kx_{1,k}, x_{1,k} \leq 2C$*, since its new payoff,* $(k-1)2C$*, would be smaller than its current payoff,* $(2k-1)C$*. Thus, the price of anarchy is at least*

$$\frac{C + kC}{2C} = \frac{k+1}{2},$$

which is unbounded as $k \to \infty$*.*

Remark 2. *In this example, we illustrate another situation of market manipulation. While a Nash equilibrium always exists, it can be arbitrarily inefficient. This is due to the fact that generator* 1 *possesses the market power. However, the market power index NMRS is not informative here, which is zero for each generator.*

5. Market Efficiency

On the other hand, the LMP mechanism may work well. We propose two sufficient conditions under either of which there exists an efficient Nash equilibrium. In the game-theoretic language, the price of stability (defined as the ratio between the cost of the best equilibrium and the socially optimal cost) is equal to 1.

5.1. Congestion-Free Condition

Definition 1 (Congestion-Free Condition)**.** *No branch power flow constraint (1c) is binding at the economic dispatch* $(x^{**}, y^{**}, \theta^{**})$.

Lemma 1. *Under the congestion-free condition, all the LMPs are equal at the economic dispatch.*

Proof. Under the congestion-free condition, $\mu_{ij}^{**} = 0$ for all (i, j). From the KKT conditions (2), we have

$$\sum_j B_{ij}(\lambda_i^{**} - \lambda_j^{**}) = 0, \ \forall i.$$

Let $\mathcal{I} = \arg\max_i \lambda_i^{**}$ be the set of buses with the largest LMPs. For each $i \in \mathcal{I}$, since $\lambda_i^{**} \geq \lambda_j^{**}$ and $B_{ij} \geq 0$ for all j, there must be $\lambda_j^{**} = \lambda_i^{**}$ for j connected to i such that $B_{ij} > 0$. It follows by the connectedness of the network that all the buses belong to $\mathcal{I}$. That is, $\lambda_i^{**} = \lambda_1^{**}$ for all i. □

It is immediate to prove by construction the existence of efficient Nash equilibria under the congestion-free condition, which also tells how to compute such equilibria directly. In the following, let Q be a large enough constant.

Theorem 1. *Under the congestion-free condition, there exists an efficient Nash equilibrium in the economic dispatch game.*

Proof. By Lemma 1, $\lambda_i^{**} = \lambda_1^{**}$ for all i. Let $\hat{c}^*$ be an offer profile where $\hat{c}_n^*(x_n) = \lambda_1^{**} x_n, x_n \leq Q$ for all n. Clearly, $(x^{**}, y^{**}, \theta^{**}, \lambda^{**}, \mu^{**})$ is also a solution to (3). It remains to show that $\hat{c}^*$ is a Nash equilibrium.

Consider generator n located at bus i. From the KKT conditions, $x_n^{**} \in \arg\max_{x_n} \lambda_1^{**} x_n - c_n(x_n)$. Its current payoff is $\pi_n^* = \lambda_1^{**} x_n^{**} - c_n(x_n^{**})$. Suppose that it changes its offer to $\hat{c}_n$, resulting a new dispatch $(\hat{x}, \hat{y}, \hat{\theta}, \hat{\lambda}, \hat{\mu})$ given

$(\hat{c}_n, \hat{c}^*_{-n})$. In light of $\hat{c}^*_{-n}$, we have $\hat{\lambda}_i \leq \lambda^{**}_1$. Its new payoff would be

$$\begin{aligned}\hat{\pi}_n &= \hat{\lambda}_i \hat{x}_n - c_n(\hat{x}_n)\\ &\leq \lambda^{**}_1 \hat{x}_n - c_n(\hat{x}_n)\\ &\leq \lambda^{**}_1 x^{**}_n - c_n(x^{**}_n)\\ &= \pi^*_n.\end{aligned}$$

Thus, it has no incentive to deviate. This proves that the constructed offer profile is a Nash equilibrium. □

Remark 3. *Since each $\hat{c}^*_n$ is linear (with a large enough cap Q), it is essentially one-segment so that the value of s^-_n does not matter. On the other hand, if we set $s^-_n = x^{**}_n$ for all n, this provides a suggested dispatch point for the ISO for tie-breaking purposes. To put it another way, if we further set $r^+_n = \lambda^{**}_1 + \delta$ with a small enough $\delta > 0$ for all n, then we obtain an efficient ϵ-Nash equilibrium.*

5.2. Monopoly-Free Condition

Definition 2 (Monopoly-Free Condition)**.** *There are at least two, or no generators at each bus.*

This condition is easier to use than the congestion-free condition, since we only need to know the placement of the generators. The constructive proof is similar as before.

Theorem 2. *Under the monopoly-free condition, there exists an efficient Nash equilibrium in the economic dispatch game.*

Proof. Let $\hat{c}^*$ be an offer profile where $\hat{c}^*_n(x_n) = \lambda^{**}_i x_n, x_n \leq Q$ for all $n \in N_i$. Then $(x^{**}, y^{**}, \theta^{**}, \lambda^{**}, \mu^{**})$ is also a solution to (3). It remains to show that $\hat{c}^*$ is a Nash equilibrium.

Consider generator n located at bus i. From the KKT conditions, $x^{**}_n \in \arg\max_{x_n} \lambda^{**}_i x_n - c_n(x_n)$. Its current payoff is $\pi^*_n = \lambda^{**}_i x^{**}_n - c_n(x^{**}_n)$. Suppose that it changes its offer to $\hat{c}_n$, resulting a new dispatch $(\hat{x}, \hat{y}, \hat{\theta}, \hat{\lambda}, \hat{\mu})$ given $(\hat{c}_n, \hat{c}^*_{-n})$. Under the monopoly-free condition, there is at least one another generator m located at the same bus i, whose offer is $\hat{c}^*_m(x_m) = \lambda^{**}_i x_m, x_m \leq Q$. So we have $\hat{\lambda}_i \leq \lambda^{**}_i$. Generator n's new payoff would be

$$\begin{aligned}\hat{\pi}_n &= \hat{\lambda}_i \hat{x}_n - c_n(\hat{x}_n)\\ &\leq \lambda^{**}_i \hat{x}_n - c_n(\hat{x}_n)\\ &\leq \lambda^{**}_i x^{**}_n - c_n(x^{**}_n)\\ &= \pi^*_n.\end{aligned}$$

Thus, it has no incentive to deviate. This proves that the constructed offer profile is a Nash equilibrium. □

Example 3. *In Example 2, the monopoly-free condition holds, so that an efficient Nash equilibrium exists:*

$$\hat{c}^*_{1,k}(x_{1,k}) = x_{1,k},\ \ x_{1,k} \le 2C,$$
$$\hat{c}^*_{2,k}(x_{2,k}) = kx_{2,k},\ \ x_{2,k} \le 2C,$$
$$\hat{c}^*_{3,k}(x_{3,k}) = kx_{3,k},\ \ x_{3,k} \le 2C,$$
$$\hat{c}^*_{4,k}(x_{4,k}) = x_{4,k},\ \ x_{4,k} \le 2C,$$

which induces the outcome

$$(x^*_{1,k}, x^*_{2,k}, x^*_{3,k}, x^*_{4,k}) = (2C, 0, 0, 0),$$
$$(\lambda^*_{1,k}, \lambda^*_{2,k}) = (1, 1).$$

Remark 4. *The intuition of the two sufficient conditions is to limit the exercise of market power. On the other hand, neither of them is necessary for the existence of Nash equilibria, whether efficient or not. That said, they may be used together for a larger class of scenarios (see Section 8 for an example).*

As a side motivation of our work, the offer format may have a significant effect on the equilibrium outcomes. It has been shown that different parametrizations of affine offer curves lead to different outcomes [21]. We further examine the case of piecewise constant offer curves.

The following example shows that while piecewise constant offer curves yield an efficient Nash equilibrium, affine offer curves may not do so.

Example 4. *Consider two identical generators located at a single bus, with cost functions $c_n(x_n) = x_n^2$ for $n = 1, 2$. The inverse demand function is $p(y) = 2-y$. Since both the congestion-free and the monopoly-free conditions hold, an efficient Nash equilibrium exists with piecewise linear reported costs:*

$$\hat{c}^*_1(x_1) = x_1,\ \ x_1 \le 1,$$
$$\hat{c}^*_2(x_2) = x_2,\ \ x_2 \le 1,$$

which induces the outcome

$$(x^*_1, x^*_2, y^*) = (0.5, 0.5, 1),\ \ \lambda^* = 1.$$

Now consider quadratic reported costs which take the following form:

$$\hat{c}_n(x_n) = \gamma_n x_n^2.$$

Suppose that there exists an efficient Nash equilibrium (γ_1, γ_2). There must be $\gamma_1 = \gamma_2 = 1$. Then either of them has an incentive to deviate. For example, given $\gamma_2 = 1$, the best response of generator 1 is $\gamma_1 = 4/3$. Thus, there is no efficient Nash equilibrium with quadratic reported cost.

6. Economic Dispatch Game with Cournot offers

We now study the economic dispatch game with Cournot offers. Note that the pricing mechanism is still the LMP mechanism.

In this game, each generator n submits a scalar offer $x_n \geq 0$ which specifies the quantity to be generated. Let $x = (x_1, \ldots, x_N)$ be the offer profile. Then the ISO solves the following problem:

$$\underset{y,\theta}{\text{maximize}} \quad \sum_i v_i(y_i) \tag{4a}$$

$$\text{subject to} \quad \sum_{n \in N_i} x_n - y_i = \sum_j B_{ij}(\theta_i - \theta_j), \ \forall i, \tag{4b}$$

$$B_{ij}(\theta_i - \theta_j) \leq C_{ij}, \ \forall (i,j), \tag{4c}$$

$$x_n \geq 0, \ \forall n, \tag{4d}$$

$$y_i \geq 0, \ \forall i. \tag{4e}$$

Compared with (1), x_n's are the inputs of (4) instead of the decision variables. Let y^* be the primal optimal solution to (4). The LMP at each bus i is given by $\lambda_i^*(x)$, the optimal dual variable associated with the bus power balance equation (4b). Thus, the payoff of generator n located at bus i is given by

$$\pi_n(x) = \lambda_i^*(x) x_n - c_n(x_n).$$

The proposed game is different from the one in [22], in which the ISO also acts as a strategic agent who moves simultaneously with the generators. Since the action set of the ISO is constrained by the generators' actions, the generalized Nash equilibrium is adopted as the solution concept. Our model is more realistic, in which the ISO determines the dispatch after the generators submit the offers. In the next, we establish the existence of Nash equilibria under certain mild assumptions. This result is similar to that in [22].

Assumption 1. *For each i, the inverse demand function $p^i(y_i)$ is strictly decreasing on $[0, D_i]$ with $p^i(D_i) = 0$ for some constant $D_i > 0$.*

Assumption 2. *For any $x \neq \mathbf{0}$, the dual optimal solution $\lambda^*(x)$ is unique. For $x = \mathbf{0}$ (so that $y^* = \mathbf{0}$), we set $\lambda_i^* = \max_j p^j(0)$ for all i.*

Assumption 2 eliminates degenerate dual optimal solutions to ensure that λ^* is a function of x, and is continuous at $x = \mathbf{0}$.

Lemma 2. *There exists some constant $M > 0$ such that in any Nash equilibrium x^*, $x_n^* \leq M$ for all n.*

Proof. Let $M = \sum_i D_i$. Suppose that $x_n > M$ for some $n \in N_i$. Then $\lambda_i^*(x) = 0$, so that $\pi_n < 0$. Thus, x cannot be a Nash equilibrium. □

Lemma 2 implies that the strategy space of each generator can be restricted to a compact set $[0, M]$.

Lemma 3. *In any Nash equilibrium, $y_i^* \leq D_i$ for all i.*

Proof. Suppose $y_i^* > D_i$ for some i. Then $\pi_i^* = 0$, which implies that there must be some generator n who can reduce x_n to get a higher payoff. Thus, $y_i^* > D_i$ cannot happen in a Nash equilibrium. □

Lemma 3 shows that the demand in equilibrium is bounded, so that the objective function $\sum_i v_i(y_i)$ is strictly concave in y.

To study the relationship between $\lambda^*(x)$ and x, we consider the (partial) Lagrangian:

$$L(y, \theta, \lambda) = \sum_i \left(v_i(y_i) + \lambda_i \left(\sum_{n \in N_i} x_i - y_i - \sum_j B_{ij}(\theta_i - \theta_j) \right) \right),$$

with $\mathbf{dom}\, L = \mathcal{Y} \times \Theta \times \Lambda$, where

$$\begin{aligned} \mathcal{Y} &= \{y | 0 \leq y_i \leq D_i, \forall i\}, \\ \Theta &= \{\theta | B_{ij}(\theta_i - \theta_j) \leq C_{ij}, \forall (i,j)\}, \\ \Lambda &= \{\lambda | 0 \leq \lambda_i \leq \max_j p^j(0), \forall i\}. \end{aligned}$$

The dual function $g : \Lambda \to \mathbb{R}$ is defined as the maximum value of the Lagrangian over (y, θ):

$$g(\lambda) = \max_{y \in \mathcal{Y}, \theta \in \Theta} L(y, \theta, \lambda) = h(\lambda) + \sum_i \lambda_i \sum_{n \in N_i} x_i,$$

where

$$h(\lambda) = \max_{y \in \mathcal{Y}} \sum_i (v_i(y_i) - \lambda_i y_i) + \max_{\theta \in \Theta} \left(-\sum_i \lambda_i \sum_j B_{ij}(\theta_i - \theta_j) \right).$$

Lemma 4. *$h(\lambda)$ is strictly convex and continuously differentiable on Λ.*

Proof. $h(\lambda)$ (as well as $g(\lambda)$) is convex, since it is the pointwise maximum of a family of affine functions of λ. By Danskin's theorem [23], it can be shown that $h(\lambda)$ is continuously differentiable on Λ.

To prove the strict convexity, it is enough to show that for $\lambda \neq \hat{\lambda}$, the corresponding maximizers (y, θ) and $(\hat{y}, \hat{\theta})$ are not equal. Suppose $(y, \theta) = (\hat{y}, \hat{\theta})$. Then $x = \hat{x}$, which means $\lambda = \hat{\lambda}$ by Assumption 2. □

Now we consider the dual problem:

$$\min_{\lambda \in \Lambda} h(\lambda) + \sum_i \lambda_i \sum_{n \in N_i} x_i.$$

The first order condition gives

$$\frac{\mathrm{d}h}{\mathrm{d}\lambda_i} = -\sum_{n \in N_i} x_i, \ \forall i.$$

Lemma 5. *λ is continuous in x.*

Proof. Since h is continuously differentiable, ∇h is continuous in λ. Since h is strictly convex, ∇h is invertible. Therefore, λ is continuous in x. $\square$

Lemma 6. *λ_i is decreasing in x_n for all i and for all n.*

Proof. $\mathrm{d}h/\mathrm{d}\lambda_i$ is decreasing in x_n for $n \in N_i$. Since h is convex, $\mathrm{d}h/\mathrm{d}\lambda_j$ is increasing in λ_j for all j. Thus, λ_j is decreasing in x_n for all j, whether j equals i or not. $\square$

Theorem 3. *Under Assumption 1 and 2, there exists a Nash equilibrium in the economic dispatch game with Cournot offers.*

Proof. The payoff of generator n located at bus i is given by

$$\pi_n(x) = \lambda_i^*(x) x_n - c_n(x_n).$$

By Lemma 5, π_n is continuous in x, which implies nonempty, closed-graph reaction correspondences. By Lemma 6, it can be shown that π_n is quasi-concave in x_n, which implies the reaction correspondences are convex-valued. It follows from Kakutani's fixed point theorem that a Nash equilibrium exists [24]. $\square$

7. Marginal Contribution Pricing Mechanism

We have studied the LMP-based economic dispatch game with various offer formats. As an alternative, we propose the MCP mechanism, in which each generator is paid a single amount of its marginal contribution, instead of a unit price of the locational marginal cost as in the LMP mechanism.

The proposed MCP mechanism is adapted from the Vickrey-Clarke-Groves (VCG) mechanism, a canonical mechanism in the mechanism design theory that implements efficient allocations in dominant strategies [25]. However, the standard VCG mechanism does not apply directly, since we require a finite-dimensional (and preferably low-dimensional) offer format while the true cost function can be infinite-dimensional.

In the MCP mechanism, we still use the realistic offer format, in which the reported cost is a two-segment piecewise linear function. The dispatch rule is also given by (3). We only change the payment rule, so that the payoff of generator m is

$$\pi_m = w_m - c_m(x_m), \tag{5}$$

where w_m is the payment made to generator m. Let (x^{-m}, y^{-m}) be the dispatch when generator m is excluded (so that $x_m^{-m} = 0$). Then w_m is given by

$$w_m = \left(\sum_i v_i(y_i^*) - \sum_{n \neq m} \hat{c}_n(x_n^*) \right) - \left(\sum_i v_i(y_i^{-m}) - \sum_{n \neq m} \hat{c}_n(x_n^{-m}) \right), \tag{6}$$

which is the (positive) externality that generator m imposes on the other generators by its participation.

Theorem 4. *There exists an efficient Nash equilibrium in the MCP-based economic dispatch game.*

Proof. Let $\hat{c}^*$ be a bid profile where $\hat{c}^*_n(x_n) = \lambda^{**}_i x_n, x_n \leq Q$ for all $n \in N_i$. Then $(x^{**}, y^{**}, \theta^{**}, \lambda^{**}, \mu^{**})$ is also a solution to problem (3). It remains to show that $\hat{c}^*$ is a Nash equilibrium.

Consider generator m located at bus i. Its current payoff is

$$\pi^*_m = \left(\sum_i v_i(y^{**}_i) - \sum_{n \neq m} \hat{c}^*_n(x^{**}_n)\right) - \left(\sum_i v_i(y^{-m}_i) - \sum_{n \neq m} \hat{c}^*_n(x^{-m}_n)\right) - c_m(x^{**}_m).$$

Suppose that it changes its offer to $\hat{c}_m$, resulting a new dispatch $(\hat{x}, \hat{y}, \hat{\theta}, \hat{\lambda}, \hat{\mu})$ given $(\hat{c}_m, \hat{c}^*_{-m})$. Its new payoff would be

$$\hat{\pi}_m = \left(\sum_i v_i(\hat{y}_i) - \sum_{n \neq m} \hat{c}^*_n(\hat{x}_n)\right) - \left(\sum_i v_i(\hat{y}^{-m}_i) - \sum_{n \neq m} \hat{c}^*_n(\hat{x}^{-m}_n)\right) - c_m(\hat{x}_m).$$

So its payoff changes by

$$\begin{aligned}
\hat{\pi}_m - \pi^*_m &= \left(\sum_i v_i(\hat{y}_i) - \sum_{n \neq m} \hat{c}^*_n(\hat{x}_n)\right) - c_m(\hat{x}_m) \\
&\quad - \left(\sum_i v_i(y^{**}_i) - \sum_{n \neq m} \hat{c}^*_n(x^{**}_n)\right) + c_m(x^{**}_m) \\
&\leq \hat{c}^*_m(\hat{x}_m) - \hat{c}^*_m(x^{**}_m) + c_m(x^{**}_m) - c_m(\hat{x}_m) \\
&= \lambda^{**}_i \hat{x}_m - \lambda^{**}_i x^{**}_m + c_m(x^{**}_m) - c_m(\hat{x}_m) \\
&\leq 0.
\end{aligned}$$

The first equality follows from the fact that $(x^{-m}, y^{-m}) = (\hat{x}^{-m}, \hat{y}^{-m})$. The first inequality follows since

$$\sum_i v_i(y^{**}_i) - \sum_n \hat{c}^*_n(x^{**}_n) \geq \sum_i v_i(\hat{y}_i) - \sum_n \hat{c}^*_n(\hat{x}_n).$$

The last inequality follows since

$$x^{**}_m \in \arg\max_{x_m} \lambda^{**}_i x_m - c_m(x_m).$$

Thus, it has no incentive to deviate. This proves that the constructed offer profile is a Nash equilibrium. □

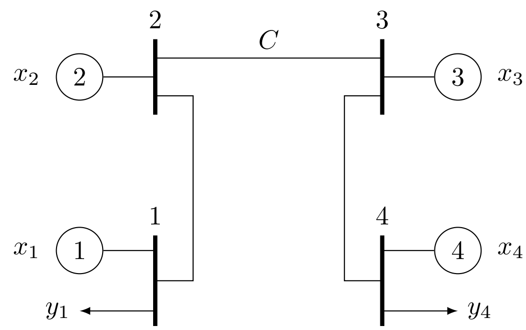


Figure 4: Example of a four-bus network which illustrates different equilibrium outcomes under different pricing mechanisms and offer formats.

8. Case Study

We present an example to illustrate the different equilibrium outcomes under different pricing mechanisms and offer formats. This example also illustrates how to apply the congestion-free and the monopoly-free conditions together in a hybrid scenario.

Consider the network as shown in Fig. 4. The network has four buses, with generator n at bus n, for $n = 1, 2, 3, 4$. All the generators have the same cost function $c_n(x_n) = x_n^2$. The inverse demand function at bus 1 is $p^1(y_1) = 1 - y_1$, and the one at bus 4 is $p^4(y_4) = 1 - 0.5y_4$. There is no demand at bus 2 and 3. The flow limit of branch 2-3 is $C = 0.05$. The other two branches have sufficiently large flow limits.

8.1. Economic Dispatch

The economic dispatch is $x_1^{**} = x_2^{**} = 0.2625, x_3^{**} = x_4^{**} = 0.325, y_1^{**} = 0.475, y_4^{**} = 0.7$, with a social welfare 0.5906.

8.2. LMP Mechanism with Piecewise Linear Reported Cost

While the sufficient conditions cannot be applied directly, we observe that branch 1-2 and branch 3-4 are always congestion-free. Thus, bus 1 and bus 2 can be viewed as a single bus, and the same for bus 3 and bus 4, in which case monopoly-free condition holds. Indeed, an efficient Nash equilibrium exists, which induces the economic dispatch. Moreover, the LMPs are $\lambda_1^* = \lambda_2^* = 0.525, \lambda_3^* = \lambda_4^* = 0.65$, and the payoffs are $\pi_1^* = \pi_2^* = 0.0689, \pi_3^* = \pi_4^* = 0.1056$.

8.3. LMP Mechanism with Cournot Offers

In this case, a Nash equilibrium is guaranteed to exist. In fact, the Nash equilibrium is unique in this example: $x_1^* = x_2^* = 0.21, x_3^* = x_4^* = 0.279$, which induces the dispatch $y_1^* = 0.37, y_2^* = 0.6071$, with a social welfare 0.5731. Clearly, the Nash equilibrium is not efficient. Moreover, the LMPs are $\lambda_1^* = \lambda_2^* = 0.63, \lambda_3^* = \lambda_4^* = 0.6965$, and the payoffs are $\pi_1^* = \pi_2^* = 0.0882, \pi_3^* = \pi_4^* = 0.1165$.

One can see that the LMPs with Cournot offers are greater than those with piecewise linear reported cost. This illustrates the exercise of market power, which leads to the inefficiency of the outcome.

8.4. MCP Mechanism with Piecewise Linear Reported Cost

In this case, an efficient Nash equilibrium always exists, which induces the economic dispatch. There is no notion of LMPs in the MCP mechanism. The payoffs are $\pi_1^* = \pi_2^* = 0.1378, \pi_3^* = \pi_4^* = 0.2113$.

The payoffs in the MCP mechanism are the highest. The interpretation is that this mechanism needs to provide enough economic incentives to ensure that an efficient Nash equilibrium always exists.

9. Conclusion

We developed a game-theoretic framework for analysis and design of market mechanisms in the context of economic dispatch. Some immediate extensions include considering demand side to be strategic as well.

We focused on the LMP mechanism with piecewise constant offer curves as in practice. On the one hand, we show that a Nash equilibrium may not exist, or that even when a Nash equilibrium exists, the price of anarchy can be arbitrarily large. On the other hand, we provide the congestion-free and the monopoly-free conditions under either of which there exists an efficient Nash equilibrium. Our findings coincide with the policy proposed in [26]: ensuring enough competition to limit the exercise of market power. The presented counterexamples also suggest that the results in the SFE literature may not apply directly to practice. Moreover, for a general transmission-constrained Cournot model in which the generators submit quantity offers, we show that there exists a Nash equilibrium under certain mild conditions.

The proposed MCP mechanism always induces an efficient Nash equilibrium, at the expense of price discrimination. Specifically, the MCP mechanism assigns a total payment to each generator, while the LMP mechanism assigns a uniform price at each bus. Like the LMP mechanism, there may be undesirable Nash equilibria in the MCP mechanism. Future work is needed to tackle these issues.

References

[1] S. Stoft, Power system economics: Designing markets for electricity, Wiley-IEEE Press, 2002.

[2] D. S. Kirschen, G. Strbac, Fundamentals of power system economics, John Wiley & Sons, 2004.

[3] McCullough Research, Congestion manipulation in ISO California (2002). URL `http://www.mresearch.com/pdfs/19.pdf`

[4] S. P. Karthikeyan, I. J. Raglend, D. Kothari, A review on market power in deregulated electricity market, International Journal of Electrical Power & Energy Systems 48 (2013) 139–147.

[5] R. D. Green, Increasing competition in the British electricity spot market, The Journal of Industrial Economics 44 (2) (1996) 205–216.

[6] R. Baldick, R. Grant, E. Kahn, Theory and application of linear supply function equilibrium in electricity markets, Journal of Regulatory Economics 25 (2) (2004) 143. doi:10.1023/B:REGE.0000012287.80449.97.

[7] J. Weber, T. Overbye, A two-level optimization problem for analysis of market bidding strategies, in: Power Engineering Society Summer Meeting, 1999. IEEE, Vol. 2, IEEE, 1999, pp. 682–687.

[8] B. Hobbs, C. Metzler, J.-S. Pang, Strategic gaming analysis for electric power systems: An MPEC approach, IEEE Transactions on Power Systems 15 (2) (2000) 638–645. doi:10.1109/59.867153.

[9] X. Hu, D. Ralph, Using EPECs to model bilevel games in restructured electricity markets with locational prices, Operations research 55 (5) (2007) 809–827.

[10] E. Hasan, F. D. Galiana, A. J. Conejo, Electricity markets cleared by merit order-Part I: Finding the market outcomes supported by pure strategy Nash equilibria, IEEE Transactions on Power Systems 23 (2) (2008) 361–371. doi:10.1109/TPWRS.2008.919238.

[11] W. W. Hogan, A market power model with strategic interaction in electricity networks, The Energy Journal 18 (4) (1997) 107.

[12] S. Borenstein, J. Bushnell, S. Stoft, The competitive effects of transmission capacity in a deregulated electricity industry, The Rand journal of economics 31 (2) (2000) 294–325.

[13] A. Ehrenmann, K. Neuhoff, A comparison of electricity market designs in networks, Operations Research 57 (2) (2009) 274–286.

[14] H. Haghighat, H. Seifi, A. R. Kian, Pay-as-bid versus marginal pricing: the role of suppliers strategic behavior, International Journal of Electrical Power & Energy Systems 42 (1) (2012) 350–358.

[15] B. Stott, J. Jardim, O. Alsac, DC power flow revisited, IEEE Transactions on Power Systems 24 (3) (2009) 1290.

[16] M. Hsu, An introduction to the pricing of electric power transmission, Utilities Policy 6 (3) (1997) 257–270.

[17] S. P. Boyd, L. Vandenberghe, Convex optimization, Cambridge University Press, 2004.

[18] California ISO, Data definitions for ISO public bid data. URL http://content.caiso.com/marketops/OASIS/pubbid/data/pbddocumentation.pdf

[19] M. B. Cain, F. L. Alvarado, Implications of cost and bid format on electricity market studies: Linear versus quadratic costs, in: Power Engineering, 2004. LESCOPE-04. 2004 Large Engineering systems Conference on, IEEE, 2004, pp. 2–6.

[20] P. Wang, Y. Xiao, Y. Ding, Nodal market power assessment in electricity markets, Power Systems, IEEE Transactions on 19 (3) (2004) 1373–1379.

[21] R. Baldick, Electricity market equilibrium models: The effect of parametrization, IEEE Transactions on Power Systems 17 (4) (2002) 1170–1176.

[22] S. Bose, D. W. Cai, S. Low, A. Wierman, The role of a market maker in networked Cournot competition, in: Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on, 2014, pp. 4479–4484. doi:10.1109/CDC.2014.7040088.

[23] D. P. Bertsekas, A. Nedić, A. E. Ozdaglar, Convex analysis and optimization, Athena Scientific, 2003.

[24] D. Fudenberg, J. Tirole, Game theory, MIT Press, 1991.

[25] A. Mas-Colell, M. Whinston, J. Green, Microeconomic theory, Oxford, 1995.

[26] J. L. Sweeney, The California electricity crisis, Hoover Institution Press, 2002.