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Stochastic Resource Auctions for Renewable Energy Integration

Wenyuan Tang
Department of Electrical Engineering
University of Southern California
wenyuan@usc.edu

Rahul Jain
EE & ISE Departments
University of Southern California
rahul.jain@usc.edu

***Abstract*—Among the many challenges of integrating renewable energy sources into the the existing grid, is the challenge of integrating renewable energy generators into the power systems economy. Electric markets currently are run in a way that participating generators must supply contracted amounts. And yet, renewable energy generators such as wind power generators cannot supply contracted amounts with certainty. Thus, alternative market architectures must be considered where there are "aggregator" entities who participate in the electricity market by buying power from the renewable energy generators, and assuming risk of any shortfall from contracted amounts. In this paper, we propose auction mechanisms that can be used by the aggregators for procuring stochastic resources, such as wind power. The nature of stochastic resources is different from classical resources in that such a resource is only available stochastically. The distribution of the generation is private information, and the system objective is to truthfully elicit such information. We introduce a variant of the VCG mechanism for this problem. We also propose a non-VCG mechanism with a contracted-payment-plus-penalty payoff structure. We show generalization of the mechanisms to general objective functions, as well as multiple winners. We then consider the setting where the generators need to fulfill any shortfall from the contracted amount by buying from the spot market.**

***Index Terms*—Renewable energy integration, smart grid, mechanism design, stochastic resource auctions.**

I. Introduction

Renewable energy will increasingly constitute a greater fraction of the energy portfolio. In fact, on April 12, 2011, the Governor of California signed legislation to require one-third of the state's electricity to come from renewable energy by 2020. Apart from the significant investment required, there are many system challenges in integrating renewable energy into the current power grid and electricity markets. These mainly arise due to the variability and unpredictability of such energy sources. For example, wind power can vary from 0 to 100 MW (at a single plant) in a matter of a couple of hours. Solar power is equally unpredictable and highly variable, while tidal power has a cyclic nature, marked by extreme peaks (during extreme events such as storms and hurricanes).

This introduces significant challenges in matching supply with demand, which varies seasonally and by time of day but is typically quite inelastic. For this purpose, "demand response" solutions are being devised where consumers are exposed to time-varying prices via smart-meters over a smart-grid infrastructure. Massively scalable energy storage solutions will have to be a part of the solution story if such a vision of a smart grid system is to succeed.

And yet, this is not all. With one-third of energy coming from renewable sources, challenges also arise in how the *electric power economy* operates. This comprises the electricity markets that operate at various timescales (spot, day-ahead, week-ahead, etc.), the transmission capacity market, the generators, the utility companies and the consumers, and pricing to them.

An important issue is how renewable energy generators such as of wind power are to participate in and integrate with the electric power economy. How can they participate in, say day-ahead electricity markets given the uncertainty about their generation for the next day? What is the right market architecture that leads to efficiency? Though the management of uncertainty in, say wind power generation seems daunting, if the market is structured the right way, enabling hundreds and thousands of geographically dispersed renewable energy generators to aggregate, then the statistical multiplexing gains can make the managing of uncertainty much easier as the variance goes down.

In this paper, we focus on the problem of aggregating power generated by renewable energy generators. Our implicit assumption is that the current (day-ahead) electricity market architecture is not changed, but new entities called "aggregators" will be allowed to enter, who will buy power from the renewable energy generators, and then sell it in the market, assuming any

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risk arising due to uncertainty in being able to supply the contracted amount. We investigate design of auction mechanisms that aggregators can use in buying power from the renewable energy generators. These auctions must be designed in such a way that it induces the generators to reveal the true distributions of the amount of power they will be able to generate the next day. This then provides the right information to the aggregators to be able to plan optimally for the risk they assume due to shortfall in meeting their generation commitment in the day-ahead electricity market.

Literature Survey: Auction design for electricity markets is a well-studied problem. However, the problem we introduce in this paper which involves stochastic resources (e.g., the electricity from renewable energy sources to be supplied the next day) is new. Almost all of economic and auction theory deals with classical goods, i.e., non-stochastic goods that can be exchanged with certainty. In contrast, if a renewable energy generator contracts to supply Q MW of power the next day, it may be able to supply that only with some probability p. With probability $1-p$, it may fail to supply the contracted amount. The auction design problem we introduce is for such stochastic goods, which has received only scant attention, if at all.

We now provide a brief overview of some relevant work on auction and market design for electricity markets though all of it deals with classical goods. Green [4] studied a linear supply function market model to investigate methods of increasing competition (which leads to reduction in dead-weight losses) in the power market in England and Wales. Baldick, et al [1] generalized Green's model by using affine functions and introducing capacity limits. Such work primarily focused on computational approaches to finding the supply function equilibrium. Johari, et al [5] proposed market mechanisms based on revealing a class of supply functions parameterized by a single scalar, which is closely related to the proportional allocation mechanism first studied by Kelly [6]. These mechanisms are designed for auctions among conventional power generators, with no uncertainty in generation. Dynamic general equilibrium models with supply friction are studied in [3].

The most related work to this paper is Bitar, et al [2]. They investigated how an independent renewable energy generator might bid optimally in a competitive electricity pool. The energy market system considered in that paper consists of a single ex-ante day-ahead forward market with an ex-post imbalance (shortfall) penalty for scheduled contract deviations. They derived analytical expressions for the optimal contracted capacity and the corresponding expected profit. They also studied the role of a variety of factors including improved forecasting, local generation, energy storage, etc. But what they considered is a decision-making problem: a single player chooses the contracted capacity to maximize his profit.

In this paper, we formulate an auction design problem for renewable energy markets. The key issue is that the generation of each renewable energy generator is a random variable. Even the player himself has no idea of the realization, but he knows the distribution. In the designed auction, we require each player to report (the parameters of) his distribution as his bid. The auctioneer then picks one or more players as the winners who have the "best" distributions. The main objective of auction design is to elicit the players' true types (i.e., the true distributions) so that the optimal social welfare can be achieved. We call such an auction a *stochastic resource auction*. This is the first work on stochastic resource auctions to the best of our knowledge.

We propose a two-stage stochastic variant of VCG, as well as a stochastic non-VCG mechanism. We demonstrate the incentive compatibility property of the two auctions designed. We also do a revenue comparison. We show generalization of the mechanisms to general objective functions, as well as multiple winners. We then consider a setting wherein the burden of the any shortfall in generation falls on the generators themselves. We propose a VCG-type mechanism for this setting, and do its equilibrium analysis.

II. Stochastic Resource Auction Design: Problem Statement

Consider N renewable energy generators, denoted by player P_i's ($i=1,\ldots,N$). Player P_i's generation for a given future time (e.g., the next day in the setting of day-ahead markets) is a random variable denoted by X_i, which is normalized so that $X_i \in \Omega := [0,1]$. All the X_i's are independent. Assume there is a type space Θ such that the distribution of X_i (for all i) can be parameterized by a K-dimensional vector $\theta_i = (\theta_i^{(1)},\ldots,\theta_i^{(K)}) \in \Theta$. We refer to θ_i as player P_i's type. Let $z=(z_1,\ldots,z_N)$ be an outcome vector, where $z_i=1$ if player P_i is a winner and $z_i=0$ otherwise. Denote the outcome space by $\mathcal{Z}$.

Definition 1. *A* stochastic social choice function (SSCF) *$f:\Theta^N \to \mathcal{Z}$ for each possible profile of agents' types $\theta=(\theta_1,\ldots,\theta_N)\in\Theta^N$ specifies the outcome $f(\theta)\in\mathcal{Z}$.*

Note that the SSCF is independent of the realization $X=(X_1,\ldots,X_N)\in\Omega^N$, because our goal is simply to maximize the expectation of some function of the re-

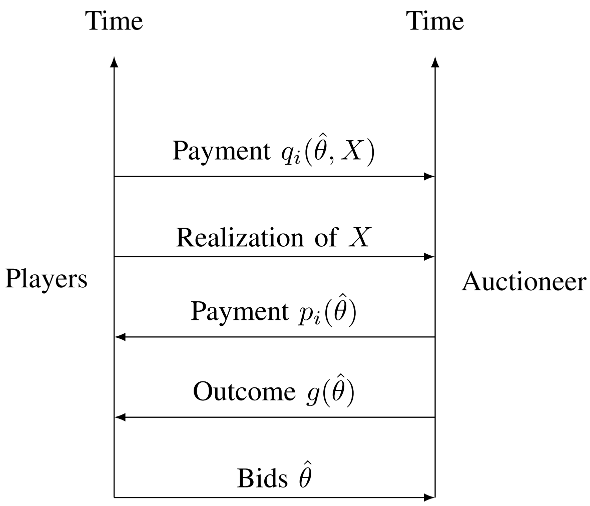

Fig. 1. The stochastic resource auction.

alizations. We next define the auction form for stochastic resources as follows.

Definition 2. *A stochastic resource auction $M = (S, g, p, q)$ is specified by*

1) *The strategy profile space $S = \times_{i=1}^{N} S_i$ from which players report their bids $s = (s_1, \ldots, s_N)$*
2) *The outcome function $g : S \to \mathcal{Z}$ which determines the winners*
3) *The payment $p_i : S \to \mathbb{R}$ made by the auctioneer to each player P_i before the realization of X*
4) *The payment $q_i : S \times \Omega^N \to \mathbb{R}$ made by each player P_i to the auctioneer after the realization of X.*

We will consider *direct stochastic resource auctions* so that the strategy space S_i for each player is the same as the type space Θ. The players report their bids $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_N)$ to the auctioneer. The auctioneer then determines the outcome $g(\hat{\theta}) = z$, indicating the winners. A payment $p_i(\hat{\theta})$ is now made to each player P_i which only depends on the bids but not on the realization of the generation. This can be interpreted as the contractual payment. Upon realizations, each player makes a payment $q_i(\hat{\theta}, X)$ to the auctioneer. This can be interpreted as the penalty that each player pays for not fulfilling some contracted amount. Note that a negative p_i or q_i indicates a reversed payment. The process of a stochastic resource auction is shown in Fig. 1.

Since the players are strategic, they may misreport their private information. Our goal is to design *incentive compatible* mechanisms that implement the SSCF in dominant strategies, i.e., that yield truthful revelation of the players' types as the *dominant strategy equilibrium.* All undefined concepts are standard, and the reader can consult [8] for their definitions.

III. Two Basic Mechanisms for Stochastic Resource Allocations

We first consider the basic scenario where there is a single winner, i.e., there exists an i' such that $z_{i'} = 1$ and $z_i = 0$ for all $i \neq i'$.

Let $F_i(\cdot)$ be the cumulative distribution function (CDF) of X_i, which is determined by the type θ_i. Let $\hat{F}_i(\cdot)$ be the CDF corresponding to the reported type $\hat{\theta}_i$.

A basic objective (represented by the SSCF) for a stochastic resource auction could be to identify the player who yields the highest expected generation (through learning the true distributions). We now propose two designs wherein it is a dominant strategy for *each player* to reveal their types truthfully. The first is a variant of the well-known VCG mechanism. The second is not, but is in some sense more natural and likely to be more acceptable to generators.

A. The Stochastic VCG (SVCG) Mechanism

We first introduce a stochastic version of the VCG mechanism. From the reported types, the associated (reported) distributions are determined, and the expected generations can be computed. We choose the player with the highest expected generation as the *winner* $P_{i'}$:

$$i' \in \arg\max_{i} \int x \,\mathrm{d}\hat{F}_i(x). \tag{1}$$

We also define the *marginal loser* $P_{i''}$ as

$$i'' \in \arg\max_{i \neq i'} \int x \,\mathrm{d}\hat{F}_i(x). \tag{2}$$

Then, the winner makes a payment $-p_{i'}$ to the auctioneer before the realization of $X_{i'}$:

$$-p_{i'} = \int x \,\mathrm{d}\hat{F}_{i''}(x). \tag{3}$$

This can be interpreted as the contractual or sign-on amount the generator pays to the auctioneer. Upon the realization of $X_{i'}$, the auctioneer makes a payment to the winner:

$$-q_{i'} = X_{i'}. \tag{4}$$

This can be interpreted as the payment for the supply that the generator actually makes to the aggregator (at price 1). Then winner's payoff is

$$U_{i'} = p_{i'} - q_{i'} = X_{i'} - \int x \,\mathrm{d}\hat{F}_{i''}(x).$$

The other players get zero payoffs.

Theorem 1. *The SVCG mechanism specified by (1)-(4) is incentive compatible.*

Due to space constraints, we omit the proofs of Theorem 1 and 2. The reader can consult the proof of Theorem 3 for the general case.

This mechanism is a stochastic variant of the Vickrey-Clarke-Groves (VCG) mechanism. The expectation of $X_{i'}$ can be viewed as the counterpart of the valuation in the standard VCG mechanism, and $\int x \,\mathrm{d}\hat{F}_{i''}(x)$ as the counterpart of the payment (externality). Unlike the classical setting, the "valuation" in our model is not intrinsic. In fact, it is also in the form of a payment. This opens the possibility of other kinds of incentive compatible mechanisms.

B. The Stochastic Shortfall Penalty (SSP) Mechanism

We now propose a non-VCG mechanism. The winner $P_{i'}$ is chosen as in (1), and the marginal loser is defined as in (2). A payment $p_{i'}$ is made in advance to the winner:

$$p_{i'} = 1. \tag{5}$$

The interpretation is that the auctioneer pays for the normalized full capacity (at price 1) before the realization of $X_{i'}$. After realization, if there is a shortfall, the contracted generator has to pay a penalty $q_{i'}$ that depends on the shortfall:

$$q_{i'} = \lambda(1 - X_{i'}), \tag{6}$$

where

$$\lambda = \frac{1}{1 - \int x \,\mathrm{d}\hat{F}_{i''}(x)}$$

can be viewed as the penalty price for the shortfall $1 - X_{i'}$. As before, the winner's payoff is

$$U_{i'} = p_{i'} - q_{i'} = 1 - \lambda(1 - X_{i'}).$$

The other players get zero payoffs.

Theorem 2. *The SSP mechanism specified by (1)-(2) and (5)-(6) is incentive compatible.*

The above mechanism is inspired by [2], which adopts a contracted-payment-plus-penalty payoff structure for a renewable energy generator. However, they have considered a decision-making problem in which the decision maker chooses the contracted capacity, while we consider an auction design problem in which players bid their distributions. Note that $\lambda \geq 1$, which is necessary for the mechanism to be incentive compatible.

It is interesting to observe a "quasi-duality" between the SVCG and the SSP mechanisms:

- Before the realization, money flows from the winner to the auctioneer in the SVCG mechanism (which depends on the second highest bid), while it flows from the auctioneer to the winner in the SSP mechanism (which is a constant).
- After the realization, money flows from the auctioneer to the winner in the SVCG mechanism (which depends on the realization), while it flows from the winner to the auctioneer in the SSP mechanism (which depends on both the second highest bid and the realization).

C. Revenue Comparison

It is useful to compare the expected revenue obtained with the two mechanisms. We assume that the auctioneer resells the acquired resource $X_{i'}$ at price 1.

Proposition 1. *The auctioneer's expected revenue in the SVCG mechanism is greater than or equal to that in the SSP mechanism.*

Proof: Since the auctioneer gets the same amount of power in both mechanisms, his revenue from resale is the same. We just need to compare his payment to the winner. Equivalently, we can compare the payment received by the winner, which is just the winner's payoff. Given that truth-telling is a dominant strategy for each player, the winner's expected payoff in the SVCG mechanism is

$$\int x \,\mathrm{d}F_{i'}(x) - \int x \,\mathrm{d}F_{i''}(x),$$

while the winner's expected payoff in the SSP mechanism is

$$\lambda \left[\int x \,\mathrm{d}F_{i'}(x) - \int x \,\mathrm{d}F_{i''}(x) \right].$$

(Readers are referred to the proof of Theorem 3 for details.) Since $\lambda \geq 1$, the winner's expected payoff in the SVCG mechanism is smaller than or equal to that in the SSP mechanism. Therefore, the auctioneer's expected revenue in the SVCG mechanism is greater than or equal to that in the SSP mechanism. ∎

IV. Generalizations of the Basic Mechanisms

A. General Objective Functions

In the previous analysis for the basic scenario, we have assumed that the social planner's objective is to contract with player $P_{i'}$ who yields the highest expected generation (see (1)). Now we generalize the social planner's objective. Assume that the social planner wants to contract with the one who yields the highest $\mathbb{E}[h(X_i)]$, where $h(\cdot)$ is a function of the random variable X_i.

We call $h(\cdot)$ the *objective function*, and we have

$$\mathbb{E}[h(X_i)] = \int h(x)\,\mathrm{d}F_i(x).$$

For example, the social planner's demand may be at a certain level $D \in [0,1]$. That is, he only needs D amount of power and does not care about how much more would be generated. The objective function then is

$$h(x) = \min\{x, D\}. \tag{7}$$

Now we propose the mechanisms for the general objective function $h(\cdot)$. As before, let i' denote the winner and i'' the marginal loser. That is,

$$i' \in \arg\max_i \int h(x)\,\mathrm{d}\hat{F}_i(x), \tag{8}$$

$$i'' \in \arg\max_{i \neq i'} \int h(x)\,\mathrm{d}\hat{F}_i(x). \tag{9}$$

Define the function space $\mathcal{H} := \{h : [0,1] \to \mathbb{R}\}$. We also define $\mathcal{H}_p$ as a subset of $\mathcal{H}$, which only contains non-negative, non-decreasing functions. We will show that the generalized SVCG mechanism is applicable to any $h(\cdot) \in \mathcal{H}$, while the generalized SSP mechanism is only valid for any $h(\cdot) \in \mathcal{H}_p$. Although $\mathcal{H}_p$ is a subset of $\mathcal{H}$, it still represents a large class of objective functions.

We now claim the following and prove the incentive compatibility of both mechanisms in an integrated framework.

Theorem 3. *(i) For any $h(\cdot) \in \mathcal{H}$, the generalized SVCG mechanism specified by (8)-(9) with*

$$-p_{i'} = \int h(x)\,\mathrm{d}\hat{F}_{i''}(x)$$

and

$$-q_{i'} = h(X_{i'}),$$

is incentive compatible.
(ii) For any $h(\cdot) \in \mathcal{H}_p$, the generalized SSP mechanism specified by (8)-(9) with

$$p_{i'} = h(1)$$

and

$$q_{i'} = \lambda[h(1) - h(X_{i'})],$$

where

$$\lambda = \frac{h(1)}{h(1) - \int h(x)\,\mathrm{d}\hat{F}_{i''}(x)},$$

is incentive compatible.

Proof: We want to show that truth-telling ($\hat{\theta} = \theta$) is a dominant strategy equilibrium.

Fix a player P_i with bid $\hat{\theta}_i = \theta_i$. We will show that he cannot be better off by reporting $\hat{\theta}_i \neq \theta_i$ for any $\hat{\theta}_{-i}$.

Suppose he is the winner. Let the marginal loser be $P_{i''}$. In the generalized SVCG mechanism, player P_i's expected payoff is

$$\begin{aligned}\mathbb{E}[U_i] &= \mathbb{E}[h(X_i)] - \int h(x)\,\mathrm{d}\hat{F}_{i''}(x) \\ &= \int h(x)\,\mathrm{d}F_i(x) - \int h(x)\,\mathrm{d}\hat{F}_{i''}(x) \\ &\geq 0,\end{aligned}$$

while in the generalized SSP mechanism, it is

$$\begin{aligned}\mathbb{E}[U_i] &= h(1) - \mathbb{E}[\lambda(h(1) - h(X_i))] \\ &= h(1) - \lambda\left[h(1) - \int h(x)\,\mathrm{d}F_i(x)\right] \\ &= \lambda\left[\int h(x)\,\mathrm{d}F_i(x) - \int h(x)\,\mathrm{d}\hat{F}_{i''}(x)\right] \\ &\geq 0.\end{aligned}$$

In both mechanisms, by changing his bid, either the outcome remains the same, or he loses the contract and then gets a zero payoff. Thus, he has no incentive to deviate.

Suppose he is a loser. Let the winner be $P_{i'}$. If player P_i changes his bid so that he outbids the winner, in the generalized SVCG mechanism, his expected payoff would be

$$\mathbb{E}[U_i] = \int h(x)\,\mathrm{d}F_i(x) - \int h(x)\,\mathrm{d}\hat{F}_{i'}(x) \leq 0,$$

while in the generalized SSP mechanism, it would be

$$\mathbb{E}[U_i] = \lambda'\left[\int h(x)\,\mathrm{d}F_i(x) - \int h(x)\,\mathrm{d}\hat{F}_{i'}(x)\right] \leq 0,$$

where

$$\lambda' = \frac{h(1)}{h(1) - \int h(x)\,\mathrm{d}\hat{F}_{i'}(x)}.$$

In both mechanisms, if he does not outbid the winner, he still gets a zero payoff. Thus, he has no incentive to deviate.

This proves the theorem. Note that the condition $h(\cdot) \in \mathcal{H}_p$ in the generalized SSP mechanism ensures that the penalty price $\lambda \geq 1$. ■

Example 1. *It is easy to verify that the objective function in (7) satisfies $h(\cdot) \in \mathcal{H}_p \subset \mathcal{H}$. Thus, both the generalized SVCG and SSP mechanisms can be used. The winner's payoff in the generalized SVCG mechanism is*

$$U_{i'} = \min\{X_{i'}, D\} - \int \min\{x, D\}\,\mathrm{d}\hat{F}_{i''}(x),$$

while that in the generalized SSP mechanism is

$$\begin{aligned} U_{i'} &= \min\{1, D\} - \lambda(\min\{1, D\} - \min\{X_{i'}, D\}) \\ &= D - \lambda(D - X_{i'})^+, \end{aligned}$$

where

$$\lambda = \frac{D}{D - \int \min\{x, D\} \, \mathrm{d}\hat{F}_{i''}(x)}, \; x^+ := \max\{x, 0\}.$$

B. Eliminating the Undesired Equilibria

It is worth pointing out that given an objective function $h(\cdot)$, any $\hat{\theta}_i$ (or equivalently, $\hat{F}_i(\cdot)$) that satisfies

$$\int h(x) \, \mathrm{d}\hat{F}_i(x) = \int h(x) \, \mathrm{d}F_i(x)$$

is a dominant strategy for player i. That is, to maximize his own payoff, player P_i does not have to report θ_i but just $\hat{\theta}_i$, as long as the above equlity holds.

Thus, while the proposed mechanisms are *dominant strategy incentive compatible*, this laxity may be considerably undesirable in some scenarios. The auctioneer may not only want to elicit the true measures for a given objective function, but also want to elicit the true types, so that this information can be used for other purposes. Hence, a stronger result would be more desirable: in addition to dominant strategy incentive compatibility, each player P_i is strictly better off by reporting $\hat{\theta}_i = \theta_i$ than any $\hat{\theta}_i \neq \theta_i$ for some $\hat{\theta}_{-i}$.

This problem can be easily fixed if the auctioneer does not announce the objective function $h(\cdot)$ to be used in the mechanism but picks it arbitrarily from $\mathcal{H}$ (or $\mathcal{H}_p$). We thus claim the following.

Proposition 2. *In the generalized SVCG (or SSP) mechanism, if the objective function $h(\cdot) \in \mathcal{H}$ (or $\mathcal{H}_p$) is chosen arbitrarily and not revealed to the bidders until the bids have been submitted, then truth-telling is a unique dominant strategy for each player.*

C. Extension to Multiple Winners

Now we consider the case of multiple winners. Instances of such extension of VCG mechanisms in the classical setting can be found in [7].

Suppose the social planner wants to contract with M ($< N$) players who yield the highest $\mathbb{E}[h(X_i)]$. We rank the players in order of the bids, i.e.,

$$\int h(x) \, \mathrm{d}\hat{F}_{i^{(1)}}(x) \geq \cdots \geq \int h(x) \, \mathrm{d}\hat{F}_{i^{(N)}}(x), \qquad (10)$$

where $(i^{(1)}, \ldots, i^{(N)})$ is a permutation of $\{1, \ldots, N\}$. Then players $P_{i^{(1)}}, \ldots, P_{i^{(M)}}$ are the winners and player $P_{i^{(M+1)}}$ is the marginal loser. We now claim the following, whose proofs are quite analogous to the previous ones and are therefore omitted.

Theorem 4. *(i) For any $h(\cdot) \in \mathcal{H}$, the M-SVCG mechanism specified by (10) with*

$$-p_{i^{(m)}} = \int h(x) \, \mathrm{d}\hat{F}_{i^{(M+1)}}(x)$$

and

$$-q_{i^{(m)}} = h(X_{i^{(m)}}),$$

for $m = 1, \ldots, M$, is incentive compatible.
(ii) For any $h(\cdot) \in \mathcal{H}_p$, the M-SSP mechanism specified by (10) with

$$p_{i^{(m)}} = h(1)$$

and

$$q_{i^{(m)}} = \lambda[h(1) - h(X_{i^{(m)}})],$$

for $m = 1, \ldots, M$, where

$$\lambda = \frac{h(1)}{h(1) - \int h(x) \, \mathrm{d}\hat{F}_{i^{(M+1)}}(x)},$$

is incentive compatible.

V. Risk-Aware Generation Assignment Auctions for Stochastic Resources

In earlier sections, we have proposed incentive compatible mechanisms for stochastic resource auctions where the auctioneer does bear the risk of not being able to acquire some capacity D from the generators. The auctioneer may be an aggregator who is committed to supplying D amount of power in an electricity market. In that case, he will have to meet any shortfall by buying power from the spot market (presumably at high prices).

In this section, we consider a related auction design problem, wherein the risk of any shortfall is borne entirely by the generators themselves. That is, the generators must buy power from the spot market and deliver it to the auctioneer if there is any shortfall. This may indeed distort the incentives of the generators who may now become conservative in their bids, thus leading to inefficiency. We call such auctions *risk-aware generation assignment auctions* since the players must take the risk of the shortfall into account.

A. Problem Statement

Consider player P_i who is required to meet a fixed demand $y_i \in [0, 1]$. As before, his generation at a future time is a random variable $X_i \in [0, 1]$ with CDF $F_i(\cdot)$ and probability density function (PDF) $f_i(\cdot)$. Let λ be a constant denoting the price of the resource in the spot market, which can also be viewed as the "penalty price". Define the (expected) cost function as

$$c_i(y_i) := \mathbb{E}[\lambda(y_i - X_i)^+] = \lambda \int_0^{y_i} (y_i - x) f_i(x) \, \mathrm{d}x,$$

which is the expected payment made by the generator to the spot market to make up for the shortfall if any, when the assigned generation is y_i.

We derive some properties of the cost function. It is easy to check the following:

$$\frac{\mathrm{d}c_i(y_i)}{\mathrm{d}y_i} = \lambda \int_0^{y_i} f_i(x)\,\mathrm{d}x = \lambda F_i(y_i),$$

$$\frac{\mathrm{d}^2 c_i(y_i)}{\mathrm{d}y_i^2} = \lambda f_i(y_i) \geq 0.$$

Thus, $c_i(y_i)$ is convex on $[0,1]$. Moreover,

$$\left.\frac{\mathrm{d}c_i(y_i)}{\mathrm{d}y_i}\right|_{y_i=0} = \lambda F_i(0) = 0, \tag{11}$$

i.e., the marginal cost at zero is zero. We also have

$$\left.\frac{\mathrm{d}c_i(y_i)}{\mathrm{d}y_i}\right|_{y_i=1} = \lambda F_i(1) = \lambda, \tag{12}$$

i.e., the marginal cost is at most λ, the spot market price.

The generation assignment auction problem is to design a mechanism to satisfy a fixed demand $D > 0$ such that the following social welfare optimization problem is solved at equilibrium:

$$\begin{aligned} \underset{y_1,\ldots,y_N,y}{\text{minimize}} \quad & \sum_i c_i(y_i) + \lambda(D-y)^+ \\ \text{subject to} \quad & y = \sum_i y_i, \\ & 0 \leq y_i \leq 1,\ \forall i. \end{aligned} \tag{13}$$

We consider two cases, depending on whether $F_i(\cdot)$ can be parameterized or not. In both cases, player P_i's (expected) payoff is defined as

$$U_i(y_i) := w_i - c_i(y_i),$$

where w_i is the payment received by player P_i from the auctioneer.

B. VCG Mechanism for Complete Parametrization

In this case, each player P_i can report the complete distribution, or equivalently, the complete cost function. Denote the reported cost function by $\tilde{c}_i(\cdot)$.

In the mechanism, the assignment $(y_1, \ldots, y_N, y)$ is a solution of the following optimization problem:

$$\begin{aligned} \underset{y_1,\ldots,y_N,y}{\text{minimize}} \quad & \sum_i \tilde{c}_i(y_i) + \lambda(D-y)^+ \\ \text{subject to} \quad & y = \sum_i y_i, \\ & 0 \leq y_i \leq 1,\ \forall i. \end{aligned}$$

Let $(y_1^{-i}, \ldots, y_{i-1}^{-i}, 0, y_{i+1}^{-i}, \ldots, y_N^{-i}, y^{-i})$ denote the assignment as a solution of the above with $y_i = 0$, i.e., when player P_i is not present. Then player P_i is paid

$$w_i = \sum_{j \neq i} \left[\tilde{c}_j(y_j^{-i}) - \tilde{c}_j(y_j)\right] + \lambda(y - y^{-i}),$$

which is the *reverse externality* that player P_i imposes on the other players by his participation.

This is just a simple variant of the standard VCG mechanism, and the incentive compatibility follows.

C. i-VCG Mechanism for Incomplete Parametrization

Now we consider the more interesting and difficult case of non-parametric $F_i(\cdot)$. In this case, it is impossible for a player to communicate the cost function exactly. Instead, we ask each player P_i to report a two-dimensional bid $b_i = (\beta_i, d_i)$, where β_i is his ask price and d_i is the maximum quantity that could be offered at price β_i.

In the mechanism, the assignment $(y_1, \ldots, y_N, y)$ is a solution of the following optimization problem:

$$\begin{aligned} \underset{y_1,\ldots,y_N,y}{\text{minimize}} \quad & \sum_i \beta_i y_i + \lambda(D-y)^+ \\ \text{subject to} \quad & y = \sum_i y_i, \\ & 0 \leq y_i \leq d_i,\ \forall i. \end{aligned}$$

Let $(y_1^{-i}, \ldots, y_{i-1}^{-i}, 0, y_{i+1}^{-i}, \ldots, y_N^{-i}, y^{-i})$ denote the assignment as a solution of the above with $d_i = 0$, i.e., when player P_i is not present. Then player P_i is paid

$$w_i = \sum_{j \neq i} \beta_j (y_j^{-i} - y_j) + \lambda(y - y^{-i}),$$

which is the reverse externality that player P_i imposes on the other players by his participation.

We call this the *i-VCG mechanism* (with incomplete parametrization). Although dominant strategy implementation is impossible in this case, we show that the i-VCG mechanism is a (weak) Nash implementation, i.e., there exists a Nash equilibrium in which the efficient assignment is achieved.

Theorem 5. *There exists an efficient Nash equilibrium in the i-VCG mechanism.*

Proof: The social welfare optimization problem (13) is a convex optimization problem that is easy to solve with the properties (11) and (12). Let the solution be $(y_1^{**}, \ldots, y_N^{**}, y^{**})$, which satisfies

$$c_1'(y_1^{**}) = \cdots = c_N'(y_N^{**}) := \mu.$$

Consider the strategy profile: $\beta_i = \mu$ and $d_i = y_i^{**}$ for all i. Clearly, it induces the efficient assignment $(y_1^{**}, \ldots, y_N^{**}, y^{**})$. It remains to show that it is a Nash equilibrium.

Consider player P_i with bid $b_i = (\mu, y_i^{**})$. His current payoff is

$$U_i(y_i^{**}) = \lambda y_i^{**} - c_i(y_i^{**}).$$

If he changes his bid to decrease his contracted capacity y_i^{**} by a $\delta > 0$, then the others' assignment does not change but his payoff becomes

$$U_i(y_i^{**} - \delta) = \lambda(y_i^{**} - \delta) - c_i(y_i^{**} - \delta).$$

So his payoff changes by

$$\begin{aligned} & U_i(y_i^{**} - \delta) - U_i(y_i^{**}) \\ = \; & -\lambda\delta - c_i(y_i^{**} - \delta) + c_i(y_i^{**}) \\ \leq \; & -c_i'(y_i^{**})\delta - c_i(y_i^{**} - \delta) + c_i(y_i^{**}) \\ \leq \; & 0. \end{aligned}$$

If he changes his bid to increase his contracted capacity y_i^{**} by a $\delta > 0$, then y^{**} does not change but the assignment of some player P_j $(j \neq i)$ changes to y_j^*. Player P_i's payoff becomes

$$\begin{aligned} & U_i(y_i^{**} + \delta) \\ = \; & \sum_{j \neq i} \mu(y_j^{**} - y_j^*) + \lambda \left(y^{**} - \sum_{j \neq i} y_j^{**} \right) \\ & -c_i(y_i^{**} + \delta) \\ = \; & \mu\delta + \lambda y_i^{**} - c_i(y_i^{**} + \delta) \\ = \; & c_i'(y_i^{**})\delta + \lambda y_i^{**} - c_i(y_i^{**} + \delta). \end{aligned}$$

So his payoff changes by

$$\begin{aligned} & U_i(y_i^{**} + \delta) - U_i(y_i^{**}) \\ = \; & c_i'(y_i^{**})\delta - c_i(y_i^{**} + \delta) + c_i(y_i^{**}) \\ \leq \; & 0. \end{aligned}$$

Thus, player P_i has no incentive to deviate, which proves that the constructed strategy profile is a Nash equilibrium. ■

VI. CONCLUSION

In this paper, we have formulated auction design problems for auctioning stochastic resources among renewable energy generators. The mechanisms can be used by an "aggregator" who can then bid in a futures electricity market.

We have considered two alternative market architectures. In the first, the risk due to uncertain generation is assumed by the aggregator, and the generators compete for contract. The designed mechanisms are incentive compatible, in which the generators would truthfully reveal their probability distributions. This is achieved via a two-part payment, an ex ante payment (before the realization) and an ex post payment (after the realization). Such mechanisms are important and useful for the aggregator since he can now hedge against his risk.

In the second instance, the risk due to uncertain generation is assumed by the generators, and the generators compete for assignment. If there is any shortfall, the generators are responsible for buying from the spot market and fulfilling their contracts. This can possibly skew incentives, and make the generators averse to truthfully reporting their probability distributions, as well as make achievement of social welfare optimization difficult. It turns out that this is not the case. In the parametric case, the VCG mechanism still yields incentive compatibility, while in the non-parametric case, the i-VCG mechanism is a Nash implementation (dominant strategy implementation cannot be achieved in this case).

In the future, we will consider a repeated version of stochastic resource auctions with spot market prices that vary according to a Markov process. We would also consider a double-sided market architecture wherein there are buyers (the utility companies) as well as both types of sellers (conventional as well as renewable energy generators). It is an open question whether it is even possible to design such a market with desirable equilibrium properties. If this is impossible, then this would provide regulators with a rationale to consider alternative market architectures, as well as allow for participation of newer entities who will act as "aggregators".

The presented work, along with the proposed future work, will potentially provide economic solutions for integrating renewable energy generators into smart grid networks.

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