Hierarchical Auction Mechanisms for Network Resource Allocation

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***Abstract*—Motivated by allocation of bandwidth, wireless spectrum and cloud computing services in secondary network markets, we introduce a hierarchical auction model for network resource allocation. A Tier 1 provider owns a homogeneous network resource and holds an auction to allocate this resource among Tier 2 operators, who in turn allocate the acquired resource among Tier 3 entities. The Tier 2 operators play the role of middlemen, since their utilities for the resource depend on the revenues gained from resale. We first consider static hierarchical auction mechanisms for indivisible resources. We study a class of mechanisms wherein each sub-mechanism is either a first-price or VCG auction, and show that incentive compatibility and efficiency cannot be simultaneously achieved. We also briefly discuss sequential auctions as well as the incomplete information setting. We then propose two VCG-type hierarchical mechanisms for divisible resources. The first one is composed of single-sided auctions at each tier, while the second one employs double-sided auctions at all tiers except Tier 1. Both mechanisms induce an efficient Nash equilibrium.**

***Index Terms*—Network economics, mechanism design, auctions, hierarchical models, resource allocation.**

I. Introduction

As NETWORKS have become increasingly complex, so has the ownership structure. This means that traditional models and allocation mechanisms used for resource exchange between primary owners and end-users are no longer always relevant. Increasingly, there are middlemen, operators who buy network resources from primary owners and then sell them to end-users. Although middlemen play an important role in the distribution channel by matching supply and demand, they also potentially cause inefficiencies in network resource allocation.

Consider the scenario of bandwidth allocation. Network bandwidth is primarily owned by Tier 1 ISPs (Internet Service Providers), who then sell it to various Tier 2 ISPs. Tier 2 ISPs then sell it further to Tier 3 ISPs, and so on. The presence of ISPs in the middle stages can potentially skew network resource allocation, and cause inefficiencies from a social welfare point of view. Similarly, in the case of wireless spectrum, primary users that acquire spectrum from the FCC and lease some of it to secondary users also play the role of middlemen in secondary spectrum markets. As another example, consider cloud computing services by providers such as IBM, Google, Amazon and others for enterprise end-users (e.g., enterprises having small computational or data center needs). Gartner [13] predicts that as cloud services are more widely adopted, there will be cloud service brokerages (e.g., Appirio) that will act as middlemen between providers and end-users. This raises the key question regarding what incentive compatible or efficient hierarchical mechanisms can be used in the presence of middlemen, and whether these two can be achieved together at all.

Auctions as mechanisms for network resource allocation have received considerable attention recently. Following up on the network utility model proposed by Kelly [10], Johari and Tsitsiklis showed that the Kelly mechanism (with per-link bids) can exhibit up to 25% efficiency loss [8]. This led to a flurry of activity in designing efficient network resource allocation mechanisms, including the work of Maheshwaran and Basar [14], Johari and Tsitsiklis [9], Yang and Hajek [19], Jain and Walrand [3], Jia and Caines [6] among others [1], [15]. Most of the work focused on single-sided auctions for divisible resources, and is related to the approach of Lazar and Semret [12]. Double-sided network auctions for divisible resources were developed in [3]. One of the very few to focus on indivisible network resources is Jain and Varaiya [4] which proposed a Nash implementation combinatorial double auction. This is also the only work known so far that presents an incomplete information analysis of combinatorial market mechanisms [5].

All these mechanisms either involve network resource allocation by an auctioneer among multiple buyers, or resource exchange among multiple buyers and sellers. Most of the proposed mechanisms are Nash implementations, i.e., in which truth-telling is a Nash equilibrium but not necessarily a "dominant strategy" equilibrium. In reality, however, markets for network resources often involve middlemen. Often, they enable markets that do not exist due to information asymmetries, but that can also potentially cause inefficiencies. However, models with middlemen have not been studied much, primarily due to the difficulty of designing appropriate mechanisms. Even in economic and game theory literature, the closest related model is one that involves a resale among the same set of players after an auction, in which the winners can resell the acquired resources to the losers [2].

There is indeed some game-theoretic work on network pricing in a general topology. [7] studied a network formation game where the nodes wish to form a graph to route traffic among themselves. [17] examined how transit and customer prices and quality of service are set in a 3-tier network. However, such work focused on the pricing equilibrium, and problems like mechanism design were not studied.

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In this paper, we consider a multi-tier setting. A Tier 1 provider owns a homogeneous network resource and holds an auction to allocate this resource among Tier 2 operators, who in turn allocate the acquired resource among Tier 3 entities, and so on. Each end-user has a valuation for the resource as a function of acquired capacity, while the middlemen do not have any intrinsic valuation of the resource but a quasi-valuation which depends on the revenue gained from resale. Our goal is to design *hierarchical auction mechanisms* with desirable properties.

We first consider hierarchical mechanism design for indivisible goods. We study a class of mechanisms wherein each sub-mechanism is either a first-price or VCG auction, and show that incentive compatibility and efficiency cannot be achieved simultaneously by such hierarchical mechanisms. This seems to foretell a more general impossibility of achieving both incentive compatibility and efficiency in a hierarchical setting. We then study some representative sequential hierarchical mechanisms with both complete and incomplete information settings, and again observe the difficulty of achieving incentive compatibility and efficiency simultaneously. When the network resource is divisible, we propose two VCG-type mechanisms that employ two-dimensional bids, one with single-sided sub-mechanisms at all tiers, and one with double-sided sub-mechanisms at all tiers except Tier 1. We show that both mechanisms induce an efficient Nash equilibrium.

The paper is organized as follows. We introduce the problem in Section II. In Section III, we study some hierarchical mechanisms for indivisible goods. Section IV proposes two hierarchical mechanisms for divisible goods. Section V concludes the paper.

II. Problem Statement

A. The Hierarchical Model

Consider a Tier 1 provider (e.g., the FCC or Google) who owns C units of a homogeneous network resource. Such a good can be divisible (sold in arbitrary portions of the total amount) or indivisible (sold in integral units). Assume that there are K tiers in the hierarchical network. The Tier 1 provider auctions off the resource among the Tier 2 entities, referred to as the *Tier 1 auction*. Each Tier 2 entity then auctions off the good acquired in the Tier 1 auction to the Tier 3 entities, referred to as the *Tier 2 auction*, and in general at Tier k as the *Tier k auction* (for $1 \leq k < K$). An example of a 3-tier network is shown in Fig. 1. We note that players other than the Tier 1 provider may own some of the resource. However, this does not affect their strategic considerations, and hence is ignored.

We consider the Tier 1 provider as the social planner (indexed by 0), who attempts to maximize the "social welfare", which we will shortly define. This assumption is valid when the auctioneer is a governmental agency such as the FCC, and might still be reasonable even when the provider is a profit maximizer (since the two goals are not necessarily incompatible). The entities at other tiers are strategic players (indexed by $i = 1, \ldots, N$), among which the Tier k (for $1 < k < K$) entities are regarded as the middlemen, and the Tier K entities as the end-users.

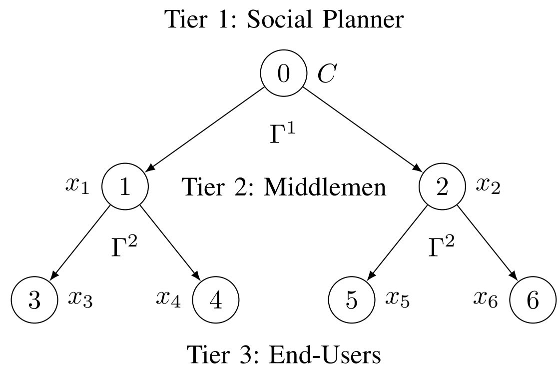

Fig. 1. An example of a 3-tier network with $N = 6$.

The hierarchical model we consider is highly stylized, and each player can acquire the resource only from its parent in the upper tier; issues like routing and peering are not taken into account. The stylized model yields concrete results that help us in gaining an insight into the problem. In fact, even in this rather simplified model, we obtain some negative results, which suggest the difficulty of hierarchical mechanism design in more general settings.

B. The Mechanism Design Framework

Let $\mathcal{T}(i)$ be the tier to which player i belongs, and $\mathtt{ch}(i)$ be the set of player i's children. Denote the capacity acquired by player i by x_i. Assume that each player i has a quasilinear utility function $u_i = v_i(x_i) - w_i$, where $v_i(\cdot)$ is his valuation function and w_i is his payment. When player i is an end-user, $v_i(\cdot)$ is intrinsic; when player i is a middleman, $v_i(x_i) = \pi_i - c_i(x_i)$, where π_i is his revenue gained from resale and $c_i(\cdot)$ is his cost function, since middlemen do not derive utilities from the resource but may incur transaction costs.

Denote $x = (x_1, \ldots, x_N)$. We define the *social welfare* $S(\cdot)$ as

$$S(x) = \sum_{i:\mathcal{T}(i)=K} v_i(x_i) - \sum_{i:1<\mathcal{T}(i)<K} c_i(x_i),$$

the difference between the aggregate valuation derived by the end-users and the aggregate cost incurred by the middlemen. This is the aggregate social surplus generated by an allocation x.

The social planner's objective is to achieve an *efficient* allocation $x^{**} = (x_1^{**}, \ldots, x_N^{**})$ that solves the social welfare optimization (SWO) problem:

$$\begin{aligned} \text{maximize} \quad & S(x) \\ \text{subject to} \quad & \sum_{j \in \mathtt{ch}(0)} x_j \leq C, \\ & \sum_{j \in \mathtt{ch}(i)} x_j \leq x_i, \ \forall i : 1 < \mathcal{T}(i) < K, \\ & x_i \geq 0, \ \forall i. \end{aligned} \tag{1}$$

The first two constraints state that the total allocation among the buyers in each auction cannot exceed the allocation acquired by their parent. The third constraint is to ensure non-negative allocations. In addition, if the resource is indivisible, x_i's should be integers.

Since the players are strategic and may misreport their private information, our goal is to design a mechanism that induces an efficient allocation maximizing the social welfare. Note that in a hierarchical setting, the social planner specifies the mechanisms to be used at all tiers which must then be used. This is quite reasonable when the social planner is, say the government, and has the power to regulate the market.

Denote the mechanism by $\Gamma = (\Gamma^1, \ldots, \Gamma^{K-1})$, in which a common sub-mechanism Γ^k (for $1 \leq k < K$) is employed in the Tier k auctions. An auction (sub-mechanism) is a single-sided auction if buyers place bids and sellers do nothing; it is a double-sided auction if buyers place buy-bids and sellers place sell-bids. In our model, there is always only one seller in each auction.

Before going further, we provide a brief discussion of the nature of the model and the difficulties of the problem.

1) The players at different tiers may bid simultaneously or sequentially, while the resources are always allocated from Tier 1 to Tier K. This suggests that one might expect similar results between the two cases under certain conditions.
2) One key difficulty is that middlemen have no intrinsic valuations of the resource. We will introduce the notion of quasi-valuation functions as their "types", which are related to the revenues gained from resale. As a result, middlemen cannot have dominant strategies (see [16] for the definition), and we will introduce a weaker notion of a dominant strategy.
3) The hierarchical mechanism is decentralized, with multiple auctions at each tier, and the social planner holds only one of the $N - M + 1$ auctions. This makes the achievement of efficiency even more difficult.
4) For divisible resources, it is impossible for a player to report an arbitrary real-valued valuation function completely. Thus, we have to restrict the bid spaces to be finite-dimensional and focus on Nash implementation.

III. Hierarchical Auctions for Indivisible Resources

When the resources are indivisible, we study a class of mechanisms wherein the common sub-mechanism at each tier is either a first-price auction or a VCG auction.

In a first-price auction (denoted by $\mathcal{F}$), the buyer with the highest bid wins the single unit good, and pays the amount of his bid to the seller. In a second-price auction, the highest bidder wins but pays only the second-highest bid. A second-price auction gives buyers an incentive to bid their true value while a first-price auction does not. A generalization of the second-price auction to multiple goods that maintains the incentive to bid truthfully is known as the Vickrey-Clarke-Groves (VCG) auction (denoted by $\mathcal{V}$). The idea is that items are assigned to maximize the social welfare; then each player pays the "externality" imposed on the other players by his participation (see [16] for more details).

Without loss of generality, we assume that middlemen have no transaction cost, since it can be directly incorporated into the valuation.[1] Before proceeding, we need to redefine some notions for the hierarchical setting.

Definition 1. *A middleman i's* quasi-valuation function $\bar{v}_i : \mathbb{Z}^+ \to \mathbb{R}^+$ *specifies his revenue from resale for each possible allocation he may acquire, when all his children report their valuation functions (for end-users) or quasi-valuation functions (for middlemen) truthfully.*

The quasi-valuation $\bar{v}_i(x_i)$ specifies the maximum that a middleman is willing to pay for an allocation x_i. Note that this would depend on how much the middleman's children are willing to pay for such an allocation. Given an allocation and a payment rule, the quasi-valuation function is well defined. Also, note the backward-recursiveness in the definition. We now define a dominant strategy as well as incentive compatibility in this new environment, both of which are weaker than the definitions in the standard setting.

Definition 2. *Given that all the players* $\mathtt{ch}(i)$ *report their valuation or quasi-valuation functions truthfully, a strategy is a* hierarchical dominant strategy *for player i if it maximizes his payoff regardless of what the others play.*

A dominant strategy yields the best payoff for a player regardless of what the others play. A player will always play such a strategy if it exists. Note that for an end-user, a strategy is a hierarchical dominant strategy if and only if it is a dominant strategy.

Definition 3. *A hierarchical mechanism is* incentive compatible *if it induces a hierarchical dominant strategy equilibrium wherein all the players report their valuation or quasi-valuation functions truthfully.*

Such equilibrium strategies will be referred to as *truth-telling* as a counterpart of the usual notion of truth-telling in the standard setting [16]. We remind the reader that a dominant strategy equilibrium is regarded as a strong solution concept of a game because it is independent of the information (or lack thereof) that a player may have about others. Thus, incentive compatible mechanisms are regarded as very desirable.

We mainly focus on static auctions in a complete information environment. Later, we will also consider sequential auctions and the incomplete information setting, and through a case study show how equilibria for these settings can be derived.

A. Static Auctions with Complete Information

We first study the hierarchical extension of the first-price auction, which we call the *hierarchical first-price mechanism*, in which $\Gamma^1 \in \{\mathcal{F}, \mathcal{V}\}$, $\Gamma^2 = \cdots = \Gamma^{K-1} = \mathcal{F}$, i.e., the Tier 1 sub-mechanism is a first-price or VCG auction, while all the others are first-price auctions. We show the existence of an efficient ϵ-Nash equilibrium in this mechanism.

Proposition 1. *Assume the social planner and each middleman have at least two children, i.e., the outdegree of each non-terminal node is at least two. Suppose a single indivisible good*

[1] We will consider cost functions for divisible resources though this elaboration is still not crucial.

is to be allocated.[2] *In the hierarchical first-price mechanism, there exists an efficient ϵ-Nash equilibrium.*

Proof: We construct such a strategy profile as follows. Suppose in the efficient allocation, the single good is transferred in this way: $i_1 = 0 \to i_2 \to i_3 \to \cdots \to i_K$, where $\mathcal{T}(i_k) = k$ for all $1 < k \leq K$. Consider the strategy profile:

$$b_i = \begin{cases} v_{i_K}, & i = i_2, \ldots, i_K, \\ v_{i_K} - \epsilon, & \text{otherwise.} \end{cases}$$

Note that v_{i_K} is the valuation (a scalar in this case) of the winning end-user. Clearly, this profile induces the efficient allocation that solves the SWO problem (1). It is also easy to check that it is an ϵ-Nash equilibrium: no one can gain more than ϵ by unilaterally deviating from his strategy. This proves the claim. ■

Since the standard first-price auction is not incentive compatible, the hierarchical extension cannot be either. Moreover, as long as a first-price auction exists as a sub-mechanism, the entire hierarchical mechanism cannot be incentive compatible, since truth-telling is a weakly dominated strategy (always achieving a zero utility). We state this as a proposition.

Proposition 2. *The hierarchical first-price mechanism is not incentive compatible. More generally, for any hierarchical mechanism Γ, if there exists some k such that $\Gamma^k = \mathcal{F}$, then Γ cannot be incentive compatible.*

We now study the hierarchical extension of the VCG auction, which we call the *hierarchical VCG mechanism*, in which $\Gamma^1 = \cdots = \Gamma^{K-1} = \mathcal{V}$, i.e., each sub-mechanism is a VCG auction. We show the incentive compatibility of this mechanism.

Proposition 3. *Suppose multiple units of an indivisible good are to be allocated. The hierarchical VCG mechanism is incentive compatible.*

Proof: According to Definition 3, we need to show that truth-telling is a hierarchical dominant strategy equilibrium. Consider the Tier $K-1$ auction. Since this is a VCG auction, truth-telling is a dominant (and hence hierarchical dominant) strategy for each player i with $\mathcal{T}(i) = K$. For the purpose of backward induction,[3] assume each player i with $\mathcal{T}(i) = k$ reports truthfully. Then the quasi-valuation function of each player i with $\mathcal{T}(i) = k-1$ can be equivalently viewed as an "intrinsic" valuation function by Definition 1. It follows that truth-telling is again a hierarchical dominant strategy for each player i with $\mathcal{T}(i) = k-1$ by Definition 2. Thus, all the players will report truthfully. ■

The fact that the hierarchical first-price mechanism is not incentive compatible, but the hierarchical VCG mechanism is, is not surprising since the non-hierarchical first-price and VCG mechanisms respectively have these properties. However, unlike the non-hierarchical VCG mechanism, efficiency may not be achieved at the hierarchical dominant strategy equilibrium.

[2]For simplicity and to understand the essence of problems that arise in design, we focus on allocating a single unit. When multiple units present no additional complications, we consider them directly.

[3]Here "backward" refers to the network topology, whereas the game itself is still static.

We prove this surprising observation in the following proposition by providing a counterexample.

Proposition 4. *The hierarchical dominant strategy equilibrium in the hierarchical VCG mechanism may not be efficient.*

Proof: We provide a non-trivial counterexample. Consider the 3-tier network in Fig. 1 with $C = 5$. With the notation $v = (v(1), v(2), v(3), v(4), v(5))$, let the valuation functions of the end-users be $v_3 = (10, 18, 24, 28, 30)$, $v_4 = (20, 25, 29, 32, 34)$, $v_5 = (15, 24, 32, 39, 45)$, $v_6 = (16, 20, 24, 27, 29)$. Given $\Gamma^2 = \mathcal{V}$, the quasi-valuation functions of the middlemen are computed as $\bar{v}_1 = (10, 13, 15, 16, 15)$, $\bar{v}_2 = (15, 13, 16, 18, 19)$. Truth-telling is a hierarchical dominant strategy equilibrium with the allocation $(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*) = (4, 1, 3, 1, 0, 1)$. However, the efficient allocation derived by (1) is $(x_1^{**}, x_2^{**}, x_3^{**}, x_4^{**}, x_5^{**}, x_6^{**}) = (2, 3, 1, 1, 2, 1)$. Thus, the hierarchical dominant strategy equilibrium is not efficient. ■

As shown above, quasi-valuation functions are not monotone in general, which suggests that it is very "unlikely" to be efficient for a hierarchical dominant strategy equilibrium in the hierarchical VCG mechanism. Moreover, though one may derive conditions on the valuation functions of the end-users under which efficiency can be achieved for simple cases (e.g., 3-tier network with a single unit), it is hard to obtain such conditions for general K-tier networks with multiple units.

A "limited" impossibility result follows immediately when we restrict our attention to first-price and VCG auctions as sub-mechanisms.

Theorem 1 (Hierarchical Impossibility). *Suppose we allocate a single indivisible good in a K-tier network ($K \geq 3$). There does not exist an incentive compatible hierarchical mechanism Γ with $\Gamma^k \in \{\mathcal{F}, \mathcal{V}\}$ (for $1 \leq k < K$) which induces an efficient hierarchical dominant strategy equilibrium.*

Proof: By Proposition 2, incentive compatibility cannot be achieved if there exists some k such that $\Gamma^k = \mathcal{F}$. On the other hand, if $\Gamma^k = \mathcal{V}$ for all k, efficiency is not guaranteed in the hierarchical dominant strategy equilibrium by Proposition 4. This proves the claim. ■

Our conjecture is that this "limited" impossibility theorem foretells a more general impossibility result in hierarchical mechanism design with arbitrary sub-mechanisms at each tier.

B. Sequential Auctions with Complete Information

In this section, we consider a setting where the resource is allocated hierarchically via a sequential auction, i.e., auctions at various tiers do not take place simultaneously but sequentially. Design of such *sequential auctions* requires the theory of *dynamic mechanism design*, which is not well developed. Thus, we study some specific dynamic mechanisms to understand sequential hierarchical mechanism design and for simplicity, focus on a 3-tier network. We define two types of sequential auctions.

- *Top-down* auction (TD): In the first stage, Tier 2 players bid simultaneously. In the second stage, after observing all the previous bids, Tier 3 players bid simultaneously. Then the allocation is realized.

- *Bottom-up* auction (BU): In the first stage, Tier 3 players bid simultaneously which are observed by all players. In the second stage, Tier 2 players bid simultaneously, and then the allocation is realized.

Recall that a player's strategy in a game is a complete contingent plan that specifies how the player will act in every contingency in which he might be called upon to move. In TD, each middleman's strategy space is the same as that in the static game (which is identical to the action space), while each end-user's strategy must specify one action for each possible set of the middlemen's bids (which are observed when the end-users bid). In BU, however, each end-user's strategy space is the same as that in the static game, while each middleman's strategy must specify one action for each possible set of the end-users' bids.

We can deduce the equilibria in the sequential auctions by a proper "modification" of the equilibria in the static auctions.

Definition 4. *A strategy profile $s = (s_1, \ldots, s_N)$ in the sequential auction is an* adaptation *of the strategy (action) profile $a = (a_1, \ldots, a_N)$ in the static auction if $s_i(\cdot) \equiv a_i$ for all i.*

That is, in an adaptation, the strategy of each player is independent of the contingency. We have the following proposition, the proof of which is trivial and thus omitted.

Proposition 5. *Assume the static hierarchical auction and the (TD or BU) sequential auction employ the same sub-mechanisms. For any Nash equilibrium in the static auction, its adaptation is also a Nash equilibrium in the sequential auction and induces the same allocation.*

Example 1. *Consider VCG mechanisms as sub-mechanisms in the 3-tier network with v_i ($\bar{v}_i$) as the valuations (quasi-valuations) of the Tier 3 (Tier 2) players. A Nash equilibrium in the static hierarchical auction is*

$$b_i = \begin{cases} \bar{v}_i, & \mathcal{T}(i) = 2, \\ v_i, & \mathcal{T}(i) = 3. \end{cases}$$

The adaptations in TD and BU are

$$b_i^{TD} = \begin{cases} \bar{v}_i, & \mathcal{T}(i) = 2, \\ v_i, \ \forall \{b_j\}_{\mathcal{T}(j)=2}, & \mathcal{T}(i) = 3, \end{cases}$$

$$b_i^{BU} = \begin{cases} \bar{v}_i, \ \forall \{b_j\}_{\mathcal{T}(j)=3}, & \mathcal{T}(i) = 2, \\ v_i, & \mathcal{T}(i) = 3, \end{cases}$$

which are respectively the Nash equilibria in the two sequential auctions.

For dynamic games, however, subgame perfect Nash equilibria are more relevant. Recall that a strategy profile is a *subgame perfect Nash equilibrium* if it induces a Nash equilibrium in every subgame of the original game. We show that an adaptation (which is always a Nash equilibrium) may not be a subgame perfect Nash equilibrium.

Example 2. *Consider the same setting as in Example 1. It is easy to check that the adaptation in TD is also a subgame perfect equilibrium. However, the adaptation in BU is not. By a slight abuse of notation, for each player i who is a middleman, let $\bar{v}_i(\{b_j\}_{\mathcal{T}(j)=3})$ be his revenue function (that depends on the end-users' bids). Then, a subgame perfect equilibrium in BU is*

$$b_i = \begin{cases} \bar{v}_i(\{b_j\}_{\mathcal{T}(j)=3}), & \mathcal{T}(i) = 2, \\ v_i, & \mathcal{T}(i) = 3. \end{cases}$$

On the equilibrium path, we have $\bar{v}_i(\{b_j\}_{\mathcal{T}(j)=3}) = \bar{v}_i(\{v_j\}_{\mathcal{T}(j)=3}) = \bar{v}_i$, the quasi-valuation function.

C. Sequential First-Price Auction with Incomplete Information

We now consider hierarchical auctions with incomplete information. Specifically, we investigate a natural extension of first-price auctions, which we call the *sequential first-price auction with incomplete information*. Note that variants of VCG auctions for the incomplete information setting would be trivial due to the incentive compatibility.

Consider a 3-tier network with a single indivisible good to be allocated. There are two middlemen, player 1 and player 2, with N_1 and N_2 children respectively. The end-users' valuations are drawn from a commonly known prior distribution, and in particular, are i.i.d. random variables uniformly distributed on the interval $[0, 1]$. The stages of the game are as follows:

1) Nature draws a valuation $v_i \sim U[0, 1]$ for each end-user i independently and reveals it to that player.
2) The end-users bid simultaneously, with $b_i = \beta_i(v_i)$ for player i.
3) Each middleman learns the bids of his own children, but not those of the others.
4) The middlemen bid simultaneously, with $b_1 = \beta_1(v_1)$ and $b_2 = \beta_2(v_2)$ respectively.
5) The allocation and the payments are determined according to the mechanism $\Gamma^1 = \Gamma^2 = \mathcal{F}$.

We look for a perfect Bayesian equilibrium of this game. As we will see, the problem can be converted into an asymmetric first-price auction in the standard setting.[4]

Let $i \in \{1, 2\}$. It can be shown that the equilibrium strategy of player i's children is $\beta(v) = \frac{N_i - 1}{N_i} v$. The prior distribution of player i's revenue is $F_i(v) = N_i v^{N_i - 1} - (N_i - 1) v^{N_i}$, with the associated probability density function $f_i(v) = N_i(N_i - 1)(1 - v) v^{N_i - 2}$. Whenever $N_1 \neq N_2$, Γ^1 is a first-price auction with asymmetric bidders, i.e., $F_1(\cdot) \neq F_2(\cdot)$. In equilibrium, we have

$$\beta_1(0) = \beta_2(0) = 0, \ \beta_1(1) = \beta_2(1). \tag{2}$$

Let $\phi_i := \beta_i^{-1}$. We obtain the first-order condition for player i:

$$\phi_i'(b) = \frac{F_i(\phi_i(b))}{f_i(\phi_i(b))} \frac{1}{\phi_j(b) - b}. \tag{3}$$

A solution to the system of differential equations (2)-(3) constitutes equilibrium strategies among the middlemen. While a closed-form expression is not available, we derive the properties of the equilibrium strategies indirectly. Assume that $N_1 > N_2$. It is easy to check that the distribution F_1 dominates F_2 in terms of the reverse hazard rate, i.e., $\frac{f_1(v)}{F_1(v)} > \frac{f_2(v)}{F_2(v)}$ for all $v \in (0, 1)$. In [11], it is proved that the "weak" player 2 bids more aggressively than the "strong" player 1, i.e.,

[4]Due to space constraints, we only provide the sketch of the derivation. Readers can refer to [11] for details of the approach.

$\beta_1(v) < \beta_2(v)$ for all $v \in (0,1)$. Clearly, this result leads to the inefficiency in the Tier 1 auction, and therefore to the inefficiency in the entire hierarchical mechanism. Again, this is a negative result for hierarchical mechanism design: efficiency is not guaranteed even when the end-users are symmetric and first-price auctions are held everywhere.

IV. Hierarchical Auctions for Divisible Resources

Often, network resources such as bandwidth and spectrum are available as (infinitely) divisible resources. We thus, now consider hierarchical mechanisms for a divisible resource. For simplicity of exposition, we focus on the 3-tier network as in Fig. 1, with some change of notation: let player i be the ith middleman and player (i,j) be the end-user who is the jth child of player i.[5] The valuation function $v_{ij}(\cdot)$ of player (i,j) is assumed to be strictly increasing, strictly concave and continuously differentiable on $[0,\infty)$, with $v_{ij}(0)=0$. The cost function $c_i(\cdot)$ of player i is assumed to be strictly increasing, strictly convex and continuously differentiable on $[0,\infty)$, with $c_i(0)=0$. The payoff of player (i,j) is $u_{ij} = v_{ij}(x_{ij}) - w_{ij}$, where w_{ij} is the payment made by player (i,j). The payoff of player i is $u_i = \pi_i - w_i - c_i(x_i)$, where π_i is player i's revenue and w_i is the payment made by player i. We define the *endogenous budget balance* condition:

$$\pi_i = \sum_j w_{ij},\ \forall i. \tag{4}$$

The social welfare optimization problem for the case of divisible resources (DIV-OPT) is

$$\begin{aligned} \text{maximize} \quad & \sum_{(i,j)} v_{ij}(x_{ij}) - \sum_i c_i(x_i) \\ \text{subject to} \quad & \sum_i x_i \le C, && [\lambda_0] \\ & \sum_j x_{ij} \le x_i,\ \forall i, && [\lambda_i] \\ & x_i, x_{ij} \ge 0,\ \forall i,\ \forall (i,j), \end{aligned} \tag{5}$$

where λ_0 and λ_i's are the corresponding Lagrange multipliers (likewise for the following). The solution of the convex optimization problem is characterized by the Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{aligned} (c_i'(x_i) + \lambda_0 - \lambda_i)x_i = 0,\ \forall i, \\ c_i'(x_i) + \lambda_0 - \lambda_i \ge 0,\ \forall i, \\ (v_{ij}'(x_{ij}) - \lambda_i)x_{ij} = 0,\ \forall (i,j), \\ v_{ij}'(x_{ij}) - \lambda_i \le 0,\ \forall (i,j), \\ \lambda_0(\textstyle\sum_i x_i - C) = 0, \\ \textstyle\sum_i x_i - C \le 0, \\ \lambda_i(\textstyle\sum_j x_{ij} - x_i) = 0,\ \forall i, \\ \textstyle\sum_j x_{ij} - x_i \le 0,\ \forall i, \\ \lambda_0, \lambda_i, x_i, x_{ij} \ge 0,\ \forall i,\ \forall (i,j). \end{aligned}$$

Our objective is to design a hierarchical mechanism that induces an efficient allocation as a solution of the DIV-OPT problem, despite the strategic behavior of the players. With divisible resources, however, it is impossible for a player to report an arbitrary real-valued valuation (or cost) function completely. Thus, the mechanism must ask each player to communicate an approximation to the function from a finite-dimensional bid space, and dominant strategy implementation cannot be achieved here. Instead, we seek a static Nash implementation in a complete information setting. We propose two VCG-type mechanisms, one single-sided and one double-sided, both of which have two-dimensional bids that specify the unit price and the quantity. Such bid spaces are natural and used in many practical scenarios.

[5] The results extend to the general K-tier network albeit the notation is more complicated.

A. Hierarchical Single-Sided VCG Mechanism

We first propose the hierarchical single-sided VCG (HSVCG) mechanism. In the Tier 1 auction, player i reports a bid $b_i = (\beta_i, d_i)$, where β_i is the bid price and d_i is the maximum quantity desired; in the ith Tier 2 auction, player (i,j) reports a bid $b_{ij} = (\beta_{ij}, d_{ij})$, where β_{ij} is the bid price and d_{ij} is the maximum quantity desired. The allocation is then determined as follows. In the Tier 1 auction, the allocation $\tilde{x} = (\tilde{x}_i, \forall i)$ is a solution of the following optimization problem (HSVCG-1):

$$\begin{aligned} \text{maximize} \quad & \sum_i \beta_i x_i \\ \text{subject to} \quad & \sum_i x_i \le C, \quad [\mu_0] \\ & x_i \le d_i,\ \forall i, \quad [\mu_i] \\ & x_i \ge 0,\ \forall i. \end{aligned} \tag{6}$$

Let $\tilde{x}^{-i} = (\tilde{x}_l^{-i}, \forall l)$ denote the allocation as a solution of the above with $d_i = 0$, i.e., when player i is not present. Then, the payment made by player i is

$$w_i = \sum_{l \ne i} \beta_l (\tilde{x}_l^{-i} - \tilde{x}_l).$$

In the ith Tier 2 auction (in which $\tilde{x}_i$ has been determined), the allocation $\bar{x}_i = (\bar{x}_{ij}, \forall j)$ is a solution of the following optimization problem (HSVCG-2):

$$\begin{aligned} \text{maximize} \quad & \sum_j (\beta_{ij} - \beta_i) x_{ij} \\ \text{subject to} \quad & \sum_j x_{ij} \le \tilde{x}_i, && [\nu_i] \\ & x_{ij} \le d_{ij},\ \forall j, && [\nu_{ij}] \\ & x_{ij} \ge 0,\ \forall j. \end{aligned} \tag{7}$$

Let $\bar{x}_i^{-j} = (\bar{x}_{ik}^{-j}, \forall k)$ denote the allocation as a solution of the above with $d_{ij} = 0$, i.e., when player (i,j) is not present. Then, the payment made by player (i,j) is

$$w_{ij} = \beta_i \sum_k (\bar{x}_{ik} - \bar{x}_{ik}^{-j}) + \sum_{k \ne j} \beta_{ik} (\bar{x}_{ik}^{-j} - \bar{x}_{ik}),$$

and player i's revenue equals exactly the total payment of his children, as in (4).

The solution of (6) is characterized by the KKT conditions:

$$\begin{aligned}(\beta_i - \mu_0 - \mu_i)x_i = 0,\ \forall i,\\ \beta_i - \mu_0 - \mu_i \le 0,\ \forall i,\\ \mu_0(\textstyle\sum_i x_i - C) = 0,\\ \textstyle\sum_i x_i - C \le 0,\\ \mu_i(x_i - d_i) = 0,\ \forall i,\\ x_i - d_i \le 0,\ \forall i,\\ \mu_0, \mu_i, x_i \ge 0,\ \forall i,\end{aligned}$$

and the solution of (7) is characterized by the KKT conditions (given a fixed i):

$$\begin{aligned}(\beta_{ij} - \beta_i - \nu_i - \nu_{ij})x_{ij} = 0,\ \forall j,\\ \beta_{ij} - \beta_i - \nu_i - \nu_{ij} \le 0,\ \forall j,\\ \nu_i(\textstyle\sum_j x_{ij} - x_i) = 0,\\ \textstyle\sum_j x_{ij} - x_i \le 0,\\ \nu_{ij}(x_{ij} - d_{ij}) = 0,\ \forall j,\\ x_{ij} - d_{ij} \le 0,\ \forall j,\\ \nu_i, \nu_{ij}, x_{ij} \ge 0,\ \forall j.\end{aligned}$$

Theorem 2. *The HSVCG mechanism induces an efficient Nash equilibrium.*

Proof: Let $x^{**} = ((x_i^{**}, \forall i), (x_{ij}^{**}, \forall (i,j)))$ be an efficient allocation that solves (5). Then given a fixed i, $v'_{ij}(x_{ij}^{**})$'s are equal and $v'_{ij}(x_{ij}^{**}) \ge c'_i(x_i^{**})$, for all j with $x_{ij}^{**} > 0$.

Consider the strategy profile: $\beta_i^* = \max_j v'_{ij}(x_{ij}^{**})$, $d_i^* = x_i^{**}$, $\beta_{ij}^* = v'_{ij}(x_{ij}^{**})$, $d_{ij}^* = x_{ij}^{**}$. It is easy to check that x^{**} is also a solution of (6) and (7). It remains to show that the constructed strategy profile is a Nash equilibrium.[6]

Consider player i, a middleman whose payoff is $u_i^* = \beta_i^* x_i^{**} - c_i(x_i^{**})$. Suppose he deviates by changing his bid to (β_i, d_i) with the resulting allocation x_i. If it is possible for him to be better off, there must be $x_i \le x_i^{**}$ (given his children's demand fixed) and $\beta_i \le \beta_i^*$ (otherwise he will get a zero revenue). Then, we have $u_i \le \beta_i^* x_i - c_i(x_i)$, and

$$\begin{aligned}u_i - u_i^* &\le \beta_i^*(x_i - x_i^{**}) - c_i(x_i) + c_i(x_i^{**})\\ &\le c'_i(x_i^{**})(x_i - x_i^{**}) - c_i(x_i) + c_i(x_i^{**})\\ &\le 0,\end{aligned}$$

where the last inequality follows from convexity and monotonicity. Thus, he has no incentive to deviate.

Consider player (i,j), an end-user whose payoff is $u_{ij}^* = v_{ij}(x_{ij}^{**}) - v'_{ij}(x_{ij}^{**})x_{ij}^{**}$. Suppose he deviates by changing his bid to (β_{ij}, d_{ij}) with the resulting allocation x_{ij}. If it is possible for him to be better off, there must be $\beta_{ij} \ge \beta_{ij}^*$ (otherwise he will get a zero allocation). Then, $u_{ij} \le v_{ij}(x_{ij}) - v'_{ij}(x_{ij}^{**})x_{ij}$, and whether $x_{ij} \ge x_{ij}^{**}$ or $x_{ij} < x_{ij}^{**}$, we always have

$$u_{ij} - u_{ij}^* = v_{ij}(x_{ij}) - v_{ij}(x_{ij}^{**}) + v'_{ij}(x_{ij}^{**})(x_{ij}^{**} - x_{ij}) \le 0,$$

where the inequality follows from concavity and monotonicity. Thus, he has no incentive to deviate. ■

[6]Due to space constraints, the KKT conditions used in the argument are not explicitly stated.

B. Hierarchical Double-Sided VCG Mechanism

In the HSVCG mechanism, all the sub-mechanisms were single-sided auctions. We now propose the hierarchical double-sided VCG (HDVCG) mechanism in which double-sided auctions are employed at all tiers except Tier 1. It seems that this mechanism provides more freedom for a middleman: besides a buy-bid, he can place an additional sell-bid. Nevertheless, it turns out that the two mechanisms are outcome-equivalent, in the sense that both mechanisms induce an efficient Nash equilibrium.

We specify the HDVCG mechanism for the 3-tier network. The Tier 1 auction is single-sided: player i reports a bid $b_i = (\beta_i, d_i)$, where β_i is the bid price and d_i is the maximum quantity desired. Each Tier 2 auction is double-sided: in the ith Tier 2 auction, player i reports a sell-bid $a_i = (\alpha_i, q_i)$, where α_i is the sell-bid price and q_i is the maximum quantity offered; player (i,j) reports a buy-bid $b_{ij} = (\beta_{ij}, d_{ij})$, where β_{ij} is the buy-bid price and d_{ij} is the maximum quantity desired. The allocation is then determined as follows. In the Tier 1 auction, the allocation $\tilde{x} = (\tilde{x}_i, \forall i)$ is a solution of the following optimization problem (HDVCG-1):[7]

$$\begin{aligned}\text{maximize}\quad & \sum_i \beta_i x_i\\ \text{subject to}\quad & \sum_i x_i \le C, \quad [\mu_0]\\ & x_i \le d_i,\ \forall i, \quad [\mu_i]\\ & x_i \ge 0,\ \forall i.\end{aligned} \tag{8}$$

Let $\tilde{x}^{-i} = (\tilde{x}_l^{-i}, \forall l)$ denote the allocation as a solution of the above with $d_i = 0$, i.e., when player i is not present. Then, the payment made by player i is

$$w_i = \sum_{l \ne i} \beta_l(\tilde{x}_l^{-i} - \tilde{x}_l).$$

In the ith Tier 2 auction (in which $\tilde{x}_i$ has been determined), the allocation $\bar{x}_i = (\bar{x}_{ij}, \forall j)$ is a solution of the following optimization problem (HDVCG-2):

$$\begin{aligned}\text{maximize}\quad & \sum_j (\beta_{ij} - \alpha_i)x_{ij}\\ \text{subject to}\quad & \sum_j x_{ij} \le \min\{\tilde{x}_i, q_i\}, \quad [\nu_i]\\ & x_{ij} \le d_{ij},\ \forall j, \quad [\nu_{ij}]\\ & x_{ij} \ge 0,\ \forall j.\end{aligned} \tag{9}$$

Let $\bar{x}_i^{-j} = (\bar{x}_{ik}^{-j}, \forall k)$ denote the allocation as a solution of the above with $d_{ij} = 0$, i.e., when player (i,j) is not present. Then, the payment made by player (i,j) is

$$w_{ij} = \alpha_i \sum_k (\bar{x}_{ik} - \bar{x}_{ik}^{-j}) + \sum_{k \ne j} \beta_{ik}(\bar{x}_{ik}^{-j} - \bar{x}_{ik}),$$

and player i's revenue is

$$\pi_i = \sum_j \beta_{ij}\bar{x}_{ij}.$$

Note that unlike HSVCG, the endogenous budget balance is

[7]Note that HDVCG-1 is identical to HSVCG-1.

not necessarily achieved in HDVCG; however, as we will see, it is ensured at the efficient Nash equilibrium we construct.

The solution of (8) is characterized by the KKT conditions:

$$\begin{aligned}(\beta_i - \mu_0 - \mu_i)x_i &= 0,\ \forall i,\\ \beta_i - \mu_0 - \mu_i &\le 0,\ \forall i,\\ \mu_0(\textstyle\sum_i x_i - C) &= 0,\\ \textstyle\sum_i x_i - C &\le 0,\\ \mu_i(x_i - d_i) &= 0,\ \forall i,\\ x_i - d_i &\le 0,\ \forall i,\\ \mu_0, \mu_i, x_i &\ge 0,\ \forall i,\end{aligned}$$

and the solution of (9) is characterized by the KKT conditions (given a fixed i):

$$\begin{aligned}(\beta_{ij} - \alpha_i - \nu_i - \nu_{ij})x_{ij} &= 0,\ \forall j,\\ \beta_{ij} - \alpha_i - \nu_i - \nu_{ij} &\le 0,\ \forall j,\\ \nu_i(\textstyle\sum_j x_{ij} - \min\{x_i, q_i\}) &= 0,\\ \textstyle\sum_j x_{ij} - \min\{x_i, q_i\} &\le 0,\\ \nu_{ij}(x_{ij} - d_{ij}) &= 0,\ \forall j,\\ x_{ij} - d_{ij} &\le 0,\ \forall j,\\ \nu_i, \nu_{ij}, x_{ij} &\ge 0,\ \forall j.\end{aligned}$$

Theorem 3. *The HDVCG mechanism induces an efficient Nash equilibrium, at which the endogenous budget balance is achieved.*

Proof: Let $x^{**} = ((x_i^{**}, \forall i), (x_{ij}^{**}, \forall (i,j)))$ be an efficient allocation that solves (5).

Consider the strategy profile: $\beta_i^* = \alpha_i^* = \max_j v'_{ij}(x_{ij}^{**})$, $d_i^* = q_i^* = x_i^{**}$, $\beta_{ij}^* = v'_{ij}(x_{ij}^{**})$, $d_{ij}^* = x_{ij}^{**}$. It is easy to check that x^{**} is also a solution of (8) and (9). Moreover, $\pi_i = \sum_j v'_{ij}(x_{ij}^{**})x_{ij}^{**} = \sum_j w_{ij}$ for all i. It remains to show that the constructed strategy profile is a Nash equilibrium.

Consider player i, a middleman whose payoff is $u_i^* = \alpha_i^* x_i^{**} - c_i(x_i^{**})$. Suppose he deviates by changing his bid to $((\beta_i, d_i), (\alpha_i, q_i))$ with the resulting allocation x_i. If it is possible for him to be better off, there must be $x_i \le x_i^{**}$ (given his children's demand fixed) and $\alpha_i \le \alpha_i^*$ (otherwise he will get a zero revenue). Then, we have $u_i \le \alpha_i^* x_i - c_i(x_i)$, and

$$\begin{aligned}u_i - u_i^* &\le \alpha_i^*(x_i - x_i^{**}) - c_i(x_i) + c_i(x_i^{**})\\ &\le c'_i(x_i^{**})(x_i - x_i^{**}) - c_i(x_i) + c_i(x_i^{**})\\ &\le 0.\end{aligned}$$

Thus, he has no incentive to deviate.

Consider player (i,j), an end-user whose payoff is $u_{ij}^* = v_{ij}(x_{ij}^{**}) - v'_{ij}(x_{ij}^{**})x_{ij}^{**}$. Suppose he deviates by changing his bid to (β_{ij}, d_{ij}) with the resulting allocation x_{ij}. If it is possible for him to be better off, there must be $\beta_{ij} \ge \beta_{ij}^*$ (otherwise he will get a zero allocation). Then, $u_{ij} = v_{ij}(x_{ij}) - v'_{ij}(x_{ij}^{**})x_{ij}$, and whether $x_{ij} \ge x_{ij}^{**}$ or $x_{ij} < x_{ij}^{**}$, we always have

$$u_{ij} - u_{ij}^* = v_{ij}(x_{ij}) - v_{ij}(x_{ij}^{**}) + v'_{ij}(x_{ij}^{**})(x_{ij}^{**} - x_{ij}) \le 0.$$

Thus, he has no incentive to deviate. ■

V. Conclusions

In this paper, we introduced a hierarchical network resource allocation model. We developed a general hierarchical mechanism design framework for such models. Such a model and framework is novel and this paper is the first work on multi-tier auctions to our best knowledge.

When the resource is indivisible, we investigated a class of mechanisms wherein each sub-mechanism is either a first-price or VCG auction. We showed that the hierarchical mechanism with a first-price or VCG auction at Tier 1, and first-price auctions at all other tiers is efficient but not incentive compatible and surprisingly, the hierarchical VCG auction mechanism is incentive compatible but not necessarily efficient. This seems to foretell a more general impossibility of achieving both incentive compatibility and efficiency in a hierarchical setting. We also studied some representative mechanisms for sequential auctions as well as the incomplete information setting, in which similar results can be obtained.

When the resource is divisible, we propose two VCG-type mechanisms. The HSVCG mechanism is composed of single-sided auctions at each tier, while the HDVCG mechanism employs double-sided auctions at all tiers except Tier 1. Both mechanisms induce an efficient Nash equilibrium. Moreover, the HSVCG mechanism always achieves endogenous budget balance, while that is ensured only at an efficient equilibrium in the HDVCG mechanism.

We note that the mechanisms we have designed can easily be extended to the setting where there are end-users even at intermediate tiers. The key results will remain unchanged for such a setting.

Another natural question is whether more general classes of incentive compatible or efficient mechanisms can be designed than those wherein the sub-mechanisms are either first-price or VCG auctions. Indeed, this is an important question. But as we show for indivisible resources, considering just these two, leads to hierarchical mechanisms that are either efficient or incentive compatible, but not both. We expect an impossibility result which claims the nonexistence of hierarchical mechanisms that are both incentive compatible and efficient. Proving such a conjecture requires new developments, which we shall consider in future work.

In future work, we will also consider more general network topologies wherein there may be more than one resource (e.g., bandwidth on multiple links, or bandwidth, storage and computation), and allow for sub-mechanism auctions with multiple sellers. Moreover, we may allow each Tier k player to participate in any of the Tier $k-1$ auctions as well.

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