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Game-Theoretic Analysis of the Nodal Pricing Mechanism for Electricity Markets

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Abstract— **Nodal pricing is a methodology for pricing electricity and managing transmission congestion that is widely used in deregulated electricity markets. We focus on the economic dispatch problem and consider the strategic behavior of both generators and consumers. For the case of no congestion, we prove the existence of an efficient Nash equilibrium; for the case of congestion, we give examples and counterexamples to demonstrate the complexity of the outcomes. As an alternative, we propose the Power Network Second Price (PNSP) mechanism that always induces an efficient Nash equilibrium.**

Index Terms— **Game theory, mechanism design, nodal pricing, smart grid.**

I. Introduction

Transmission pricing is a key element in electricity market design [1]. As an economically efficient mechanism, nodal pricing has largely been adopted in the US, Australia and New Zealand. In this paper, we apply game theory and provide a new perspective on the nodal pricing mechanism.

In practice, different transmission pricing schemes have been proposed and implemented in various markets. [2] proposes a comprehensive transmission pricing scheme, in which transmission cost is composed of two parts: transmission service cost and transmission congestion cost. The allocation of the transmission cost is based on Kirschen's tracing method. The cost of congestion and marginal losses is calculated based on locational marginal price differences, and this approach is called nodal pricing (or locational marginal pricing), which is the focus of this paper. The reader can refer to [2], [3], [4], [5] for a comprehensive analysis of nodal pricing.

The focus of this paper is on a game-theoretic analysis of electricity markets, and in particular of the nodal pricing mechanism. [2] provides a market power analysis via Nash bargaining solution. [6] studies the equilibria when power generators bid their supply functions. Although the nodal pricing mechanism has been extensively studied, the current literature is based on the assumption of a (perfectly) competitive environment of the electricity market. In reality, however, oligopoly or even monopoly (especially on the generation side) may exist at certain locations. In this case, one cannot expect those agents to reveal their true marginal costs, and hence the efficiency of the dispatch would become vacuous.

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On the other hand, most game-theoretic studies of the electricity market do not take into account the underlying topology of the power network. For example, the study of supply function equilibria [6] often focuses on a electricity market which does not involve transmission lines. In fact, constraints such as the capacity limits of transmission lines are crucial in the design and analysis of the economic mechanisms of the electricity market. Moreover, power networks are actually more complicated than other networks such as computer networks and transportation networks, due to Kirchhoff's circuit laws.

Among the few papers that study the strategic interaction in the electricity networks involving transmission lines, [7] is based on a Cournot competition model, rather than the nodal pricing mechanism. It also focuses on the strategic behavior of one agent only, assuming the markets at the other nodes are competitive. The motivation of [8] is very close to our work. However, it focuses on the computational aspects of solving the optimization problem, and fails to address the issue about the existence of Nash equilibria.

In this paper, we apply game theory and provide a comprehensive framework of studying the strategic interactions under the nodal pricing mechanism. We adopt the same bid format as the California ISO (with simplification), i.e., a linear function with a cap, while a quadratic function is often used in a supply function based model. We allow the consumers to be strategic. In comparison, the current literature uses a fixed demand function to model the demand side. Our model is based on a general power network with line flow constraints. By reformulating the economic dispatch problem as a game, we can conduct the equilibrium analysis of the nodal pricing mechanism. Our main contributions are: 1) Via counterexamples (and contrary to popular wisdom), we show the possibilities of nonexistence of Nash equilibria in the nodal pricing mechanism; 2) To address this issue, we propose the Power Network Second Price (PNSP) mechanism, and show that it always induces an efficient Nash equilibrium.

The paper is organized as follows. First, we introduce the economic dispatch problem in Section II. Based on the optimality conditions for the problem, we introduce the nodal pricing mechanism in Section III. Then we reformulate the economic dispatch problem as the economic dispatch game in Section IV. The main results are the equilibrium analysis of the game, presented in Section V. As an alternative, we propose the PNSP mechanism with desired properties in Section VI. Section VII concludes the paper.

II. The Economic Dispatch Problem

In this section, we introduce the economic dispatch problem and adopt a model similar to [9]. The concept of nodal pricing is based on the optimality conditions for this problem.

We assume a connected power network throughout the paper, which is modeled as follows. The network consists of I nodes (or buses), indexed $i = 1, \ldots, I$. Some pairs of nodes are connected by transmission lines through which power can flow. A line connecting i and j is characterized by its electrical admittance, denoted $Y_{ij} > 0$. If there is no such line, $Y_{ij} = 0$. We have $Y_{ij} = Y_{ji}$.

Let V_i be the magnitude of the voltage at node i, and θ_i be the phase angle. The real power flow over the line from i to j is equal to

$$q_{ij} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j),$$

which ignores reactive power and line losses. By our sign convention, $q_{ij} = -q_{ji}$ is positive if the power flows from i to j. Also, it is reasonable to assume that V_i's are approximately constant. Without loss of generality, we can set $V_i \equiv 1$, so that

$$q_{ij} = Y_{ij} \sin(\theta_i - \theta_j).$$

Furthermore, due to AC power flow, the economic dispatch problem is typically a nonlinear program that is difficult to solve in practice. Therefore, a DC flow model is often used as an approximation [10], by assuming that the phase angle differences $|\theta_i - \theta_j|$ are small. Therefore, we have $\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$ and

$$q_{ij} = Y_{ij}(\theta_i - \theta_j).$$

Let the capacity limit of a line connecting i and j be $C_{ij} = C_{ji} > 0$. So we have

$$q_{ij} = Y_{ij}(\theta_i - \theta_j) \le C_{ij}.$$

Assume there are N generators, indexed $n = 1, \ldots, N$, and M consumers, indexed $m = 1, \ldots, M$. Each generator n has a cost $c_n(x_n)$ as a function of its generation x_n, which is strictly increasing, strictly convex and continuously differentiable. Each consumer m has a valuation $v_m(y_m)$ as a function of its consumption y_m, which is strictly increasing, strictly concave and continuously differentiable. Moreover, denote the set of generators at node i by N_i, and the set of consumers at node i by M_i. So the net power injected into the network at node i is

$$\sum_{n \in N_i} x_n - \sum_{m \in M_i} y_m = \sum_j q_{ij} = \sum_j Y_{ij}(\theta_i - \theta_j).$$

An *economic dispatch* is an allocation of generation or consumption of each agent in the market that maximizes the *social welfare* subject to certain transmission constraints. Note that this optimization is with respect to a given set of states of the generators, in comparison with the unit commitment problem [4], which is not in the context of nodal pricing and this paper. Formally, the economic dispatch problem is the following convex program with linear constraints:

$$\begin{aligned}
\underset{x_n, y_m, \theta_i}{\text{maximize}} \quad & \sum_m v_m(y_m) - \sum_n c_n(x_n) && (1)\\
\text{subject to} \quad & \sum_{n \in N_i} x_n - \sum_{m \in M_i} y_m = \sum_j Y_{ij}(\theta_i - \theta_j), \ \forall i, && (2)\\
& Y_{ij}(\theta_i - \theta_j) \le C_{ij}, \ \forall (i,j), && (3)\\
& x_n, y_m \ge 0, \ \forall n, \forall m, && (4)
\end{aligned}$$

where (2) is the power balance equation, and (3) is the line flow constraint.

While [9] assumes a generic agent (who can behave as either a net supplier or a net demander) at each node, we assume an arbitrary number of generators and consumers at each node. For a more complete model of the economic dispatch problem (e.g., when line losses are taken into account), the reader can refer to [11].

III. The Nodal Pricing Mechanism

A nodal price is also known as a locational marginal price (LMP). It is the marginal cost of supplying the next increment of load at each node. Mathematically, it is the Lagrange multiplier for the power balance equation at each node, specified in detail below.

Associate the Lagrange multipliers λ_i with the power balance equations (2) and $\mu_{ij} \ge 0$ with the line flow constraints (3). The solution of the economic dispatch problem (1)-(4) is characterized by the Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{aligned}
[c_n'(x_n) - \lambda_i] x_n = 0, \ \forall n \in N_i, \\
c_n'(x_n) - \lambda_i \ge 0, \ \forall n \in N_i, \\
[v_m'(y_m) - \lambda_i] y_m = 0, \ \forall m \in M_i, \\
v_m'(y_m) - \lambda_i \le 0, \ \forall m \in M_i, \\
\sum_j Y_{ij}(\lambda_i - \lambda_j + \mu_{ij} - \mu_{ji}) = 0, \ \forall i, \\
\sum_{n \in N_i} x_n - \sum_{m \in M_i} y_m - \sum_j Y_{ij}(\theta_i - \theta_j) = 0, \ \forall i, \\
\mu_{ij}[Y_{ij}(\theta_i - \theta_j) - C_{ij}] = 0, \ \forall (i,j), \\
Y_{ij}(\theta_i - \theta_j) - C_{ij} \le 0, \ \forall (i,j), \\
x_n, y_m \ge 0, \ \forall n, \forall m.
\end{aligned} \tag{5}$$

Here the Lagrange multiplier λ_i is called the nodal price at node i. Intuitively, the higher the nodal price, the more difficult to deliver power to this node. The nodal price has a clear economic interpretation: if the price at each node is fixed as the nodal price, and each agent (as a price taker) chooses the amount of generation or consumption to optimize his own payoff, then the resulting dispatch is the economic dispatch.

If the line flow constraint (3) is binding (i.e., $\mu_{ij} > 0$) for some (i,j), we say *congestion* exists. There are some counterintuitive facts about nodal pricing, due to Kirchhoff's circuit laws (under DC approximation): power should flow over all possible paths, proportional to the line admittance. The reader can refer to [3], [9].

IV. Game-Theoretic Equilibrium Analysis

The nodal pricing mechanism is based on the assumption that the environment is competitive, i.e., the cost and valuation functions of the generators and the consumers can be truthfully revealed so that the independent system operator (ISO) can solve the actual economic dispatch (as opposed to one based on *reported* cost and valuation functions) and implement it by nodal pricing.

In reality, however, oligopoly or even monopoly may exist at some nodes. As was suggested in [9], one cannot expect the agents to reveal their true costs or benefits. That is, the agents may have an incentive to not reveal their private information truthfully. The main purpose of this paper is to analyze and check whether this is indeed the case, and the equilibrium outcome under nodal pricing when agents act strategically. In such a game, the reported type (or bid) can be different from the true type. In this section, we will first define the bid format, and then reformulate the problem as a game.

A. Bid Format

In the day-ahead market, generators and consumers place bids for the system operator to determine the economic dispatch as well as the nodal prices. For communication and computational tractability, the bid is usually a low dimensional function, which cannot fully represent the exact cost (or valuation) function. It is typically in the form of a piecewise linear function with multiple segments. The interpretation is that each linear segment specifies the marginal cost (or valuation) over a range of quantities for the generator (or consumer). Thus, such a bid corresponds to a piecewise linear cost (or valuation) function. According to the California ISO [12], for example, "There are 10 bid segments and 11 associated bid points. Each bid point has a generation (MW) and price (PR) value, which are paired together as MW and price coordinates."

In this paper, we adopt a simplified bid format. Each generator n is asked to report a two-dimensional bid (a_n, s_n), where a_n is the minimum unit price under which he is willing to produce, up to s_n units. Each consumer m is asked to report a two-dimensional bid (b_m, d_m), where b_m is the maximum unit price under which he is willing to buy, up to d_m units. Each bid corresponds to a linear cost (or valuation) function with a cap (i.e., the maximum capacity constraint).

B. The Economic Dispatch Game

Now we reformulate the economic dispatch problem as a game, which we call the (nodal pricing based) economic dispatch game. This reformulation is essentially to redefine the nodal pricing mechanism with respect to the bid space instead of the type space.

After the players place the bids (a_n, s_n)'s and (b_m, d_m)'s, the nodal pricing mechanism determines the dispatch as a solution of the following optimization problem:

$$\underset{x_n, y_m, \theta_i}{\text{maximize}} \quad \sum_m b_m y_m - \sum_n a_n x_n \tag{6}$$

$$\text{subject to} \quad \sum_{n \in N_i} x_n - \sum_{m \in M_i} y_m = \sum_j Y_{ij}(\theta_i - \theta_j), \ \forall i, \tag{7}$$

$$Y_{ij}(\theta_i - \theta_j) \le C_{ij}, \ \forall (i,j), \tag{8}$$

$$x_n \le s_n, \ \forall n, \tag{9}$$

$$y_m \le d_m, \ \forall m. \tag{10}$$

$$x_n, y_m \ge 0, \ \forall n, \forall m, \tag{11}$$

Since the solution of the linear program may not be unique, we break ties by maximizing the amount of generation or consumption in the lexicographic order.

Associate the Lagrange multipliers λ_i with (7), $\mu_{ij} \ge 0$ with (8), $\gamma_n \ge 0$ with (9), and $\nu_m \ge 0$ with (10). The solution of the economic dispatch game (6)-(11) is characterized by the KKT conditions:

$$\begin{aligned}
(a_n - \lambda_i + \gamma_n) x_n &= 0, \ \forall n \in N_i, \\
a_n - \lambda_i + \gamma_n &\ge 0, \ \forall n \in N_i, \\
(b_m - \lambda_i - \nu_m) y_m &= 0, \ \forall m \in M_i, \\
b_m - \lambda_i - \nu_m &\le 0, \ \forall m \in M_i, \\
\sum_j Y_{ij}(\lambda_i - \lambda_j + \mu_{ij} - \mu_{ji}) &= 0, \ \forall i, \\
\sum_{n \in N_i} x_n - \sum_{m \in M_i} y_m - \sum_j Y_{ij}(\theta_i - \theta_j) &= 0, \ \forall i, \\
\mu_{ij}[Y_{ij}(\theta_i - \theta_j) - C_{ij}] &= 0, \ \forall (i,j), \\
Y_{ij}(\theta_i - \theta_j) - C_{ij} &\le 0, \ \forall (i,j), \\
\gamma_n (x_n - s_n) &= 0, \ \forall n, \\
x_n - s_n &\le 0, \ \forall n, \\
\nu_m (y_m - d_m) &= 0, \ \forall m, \\
y_m - d_m &\le 0, \ \forall m. \\
x_n, y_m &\ge 0, \ \forall n, \forall m.
\end{aligned}$$

The payoff of generator $n \in N_i$ is

$$u_n^g = \lambda_i x_n - c_n(x_n).$$

The payoff of consumer $m \in M_i$ is

$$u_m^c = v_m(y_m) - \lambda_i y_m.$$

This completes the definition of the economic dispatch game.

We will adopt the pure Nash equilibrium as the solution concept. A Nash equilibrium of the economic dispatch game will be called *efficient* if it induces the solution of the corresponding economic dispatch problem.

V. Equilibrium Analysis

Our main results for the equilibrium analysis of the economic dispatch game are twofold: first, for the cases of no congestion, we prove the existence of efficient Nash equilibria; second, for the remaining cases (in which congestion exists somewhere), we give examples and counterexamples to demonstrate the complexity of the outcomes.

A. Case of No Congestion

First, we consider the case in which there is no congestion in the economic dispatch (i.e., the optimal solution of the economic dispatch problem). The following lemma shows the uniformity of nodal prices in this case.

Lemma 1: In the economic dispatch problem (1)-(4), if there is no congestion, then all the λ_i's are equal.

Proof: Let $\mathcal{I}$ be the set of nodes with the largest nodal prices, i.e., $\arg\max_i \lambda_i$. Since there is no congestion, $\mu_{ij} = 0$ for all (i,j), and (5) becomes

$$\sum_j Y_{ij}(\lambda_i - \lambda_j) = 0, \ \forall i.$$

Then for each $i \in \mathcal{I}$ and j connected to i (i.e., $Y_{ij} > 0$), since $\lambda_j \leq \lambda_i$, we must have $\lambda_j = \lambda_i$, or $j \in \mathcal{I}$. It follows by the connectedness of the network that all the nodes belong to $\mathcal{I}$. Therefore, all the λ_i's are equal. ■

We are ready to prove the existence of efficient Nash equilibria by construction.

Theorem 1: If there is no congestion in the optimal solution of the economic dispatch problem (1)-(4), then there exists an efficient Nash equilibrium in the corresponding economic dispatch game (6)-(11).

Proof: Let $z^* = (x_1^*, \ldots, x_N^*, y_1^*, \ldots, y_M^*)$ be an economic dispatch that solves (1)-(4), with λ_i^*'s and $\mu_{ij}^* \equiv 0$. By Lemma 1, $\lambda_i^* \equiv \lambda^*$. Consider the strategy profile: $(a_n, s_n) = (\lambda^*, x_n^*)$ for all n, $(b_m, d_m) = (\lambda^*, y_m^*)$ for all m. It is easy to check that z^* also solves (6)-(11) under this strategy profile, with $\lambda_i = \lambda^*$ and $\mu_{ij} = \gamma_n = \nu_m = 0$. It remains to show that the constructed strategy profile is a Nash equilibrium.

Consider generator n. From the KKT conditions, $x_n^* \in \arg\max_x \lambda^* x - c_n(x)$. His current payoff is $u_n^g = \lambda x_n^* - c_n(x_n^*)$. Suppose he changes his bid to $(\tilde{a}_n, \tilde{s}_n)$:

1) If $\tilde{a}_n > \lambda^*$, then $\tilde{x}_n = 0$, and therefore $\tilde{u}_n^g = 0$.
2) If $\tilde{a}_n \leq \lambda^*$, then the new nodal price will be no greater than λ^*. So $\tilde{u}_n^g \leq \lambda^* \tilde{x}_n - c_n(\tilde{x}_n) \leq u_n^g$.

Thus, he has no incentive to deviate. Similarly, we can show that each consumer m has no incentive to deviate. This proves that the constructed strategy profile is a Nash equilibrium. ■

Therefore, in the case of no congestion, one can expect a socially optimal outcome. Moreover, in the constructed efficient equilibrium, *strong budget balance* is achieved:

$$\begin{aligned}\sum_i \lambda_i^* \sum_{m \in M_i} y_m^* &= \lambda^* \sum_m y_m^* \\ &= \lambda^* \sum_n x_n^* \\ &= \sum_i \lambda_i^* \sum_{n \in N_i} x_n^*.\end{aligned}$$

In other words, there is no so-called merchandizing surplus or congestion rent (usually collected by owners of transmission lines).

B. Case of Congestion

On the other hand, if congestion exists (i.e., at least one line is constrained) in the economic dispatch, the situation is more complex. In this case, a Nash equilibrium may or may not exist. We will give three examples and counterexamples.

For simplicity of analysis, we make the additional assumptions in these examples: $v_m(0) = c_n(0) = 0$, $v_m'(0) = \infty$, $c_n'(0) = 0$.

Example 1: Consider the network as shown in Fig. 1. The network has two nodes, with one generator and one consumer at each node. The line capacity is $C > 0$. Suppose in the economic dispatch, $x_1^* = y_1^* + C$, $x_2^* = y_2^* - C$, $c_1'(x_1^*) = v_1'(y_1^*) = \lambda_1^* < c_2'(x_2^*) = v_2'(y_2^*) = \lambda_2^*$. That is, the players at node 1 have a lower marginal cost or valuation at the economic dispatch than those at node 2, so that the flow from node 1 to node 2 is constrained.

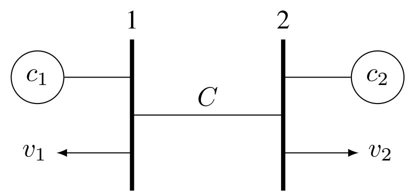

Fig. 1. A two-node network with two players of different types at each node.

Proposition 1: If C is sufficiently small, there exists an efficient Nash equilibrium in the network in Fig. 1.

Proof: Consider the strategy profile: $(a_n, s_n) = (\lambda_n^*, x_n^*)$ for $n = 1, 2$, $(b_m, d_m) = (\lambda_m^*, y_m^*)$ for $m = 1, 2$. It is easy to check that this strategy profile induces the economic dispatch. It remains to show that it is a Nash equilibrium.

Consider generator 1. His current payoff is $u_1^g = \lambda_1^* x_1^* - c_1(x_1^*)$. Suppose he changes his bid to $(\tilde{a}_1, \tilde{s}_1)$:

1) If $\tilde{a}_1 > \lambda_1^*$, then $\tilde{u}_1^g \leq \lambda_2^* C - c_1(C) < u_1^g$, given that C is sufficiently small.
2) If $\tilde{a}_1 \leq \lambda_1^*$, then the new nodal price will be no greater than λ_1^*. So $\tilde{u}_1^g \leq \lambda_1^* \tilde{x}_1 - c_1(\tilde{x}_1) \leq u_1^g$.

Thus, he has no incentive to deviate. Similarly, we can show that no other player has an incentive to deviate. This proves that the constructed strategy profile is a Nash equilibrium. ■

Example 2: Now we construct another example by changing the placement of the players: the two players at each node are of the same type, as shown in Fig. 2. Suppose in the economic dispatch, $x_1^* + x_2^* = y_1^* + y_2^* = C$, $c_1'(x_1^*) = c_2'(x_2^*) = \lambda_1^* < v_1'(y_1^*) = v_2'(y_2^*) = \lambda_2^*$.

Proposition 2: There does not exist a Nash equilibrium in the network in Fig. 2.

Proof: Suppose $(a_n, s_n)_{n=1,2}, (b_m, d_m)_{m=1,2}$ is a Nash equilibrium with the resulting dispatch $(x_n, y_m)_{n,m=1,2}$ and nodal prices $\lambda_1 \leq \lambda_2$. We derive a contradiction as follows:

1) $c_n'(x_n) \leq \lambda_1$ for $n = 1, 2$, $v_m'(y_m) \geq \lambda_2$ for $m = 1, 2$. Otherwise, that player can lower his allocation to get a better payoff.

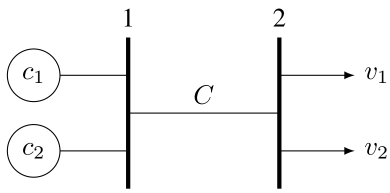

Fig. 2. A two-node network with two players of the same type at each node.

2) $c'_n(x_n) = \lambda_1$ for $n = 1, 2$. Otherwise, that generator can increase his allocation to get a better payoff.
3) Similarly, $v'_m(y_m) = \lambda_2$ for $m = 1, 2$.
4) $\lambda_1 < \lambda_2$. Otherwise, $c'_1(x_1) = c'_2(x_2) = v'_1(y_1) = v'_2(y_2)$. But this is an economic dispatch with no congestion, contradicting the assumptions.
5) Now we have $c'_1(x_1) = c'_2(x_2) = \lambda_1 < v'_1(y_1) = v'_2(y_2) = \lambda_2$. But this cannot happen in a Nash equilibrium. In fact, each player has an incentive to deviate. For example, generator 1 can change his bid to (λ_2, x_1) to get a better payoff. Therefore, there does not exist a Nash equilibrium.

■

Example 3: Consider a three-node ring network as shown in Fig. 3. There is one generator at node 1, one generator at node 2, and two consumers at node 3. The power flows are determined under the assumption that all the line admittances are equal. Only line 2-3 has a limited capacity $C > 0$. Suppose in the economic dispatch, $x_1^*/3 + 2x_2^*/3 = C$, $x_1^* + x_2^* = y_1^* + y_2^*$, $c'_2(x_2^*) = \lambda_2^* < c'_1(x_1^*) = \lambda_1^* < v'_1(y_1^*) = v'_2(y_2^*) = \lambda_3^*$.

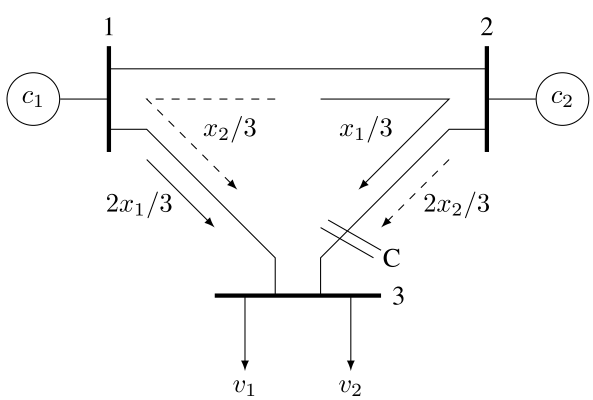

Fig. 3. A three-node ring network.

Proposition 3: There does not exist a Nash equilibrium in the network in Fig. 3.

Proof: Suppose $(a_n, s_n)_{n=1,2}, (b_m, d_m)_{m=1,2}$ is a Nash equilibrium with the resulting dispatch $(x_n, y_m)_{n,m=1,2}$ and nodal prices $\lambda_1, \lambda_2, \lambda_3$. We derive a contradiction as follows (sketch):

1) $v'_1(y_1) = v'_2(y_2) = \lambda_3$.
2) There must be either $x_1 = 0$ or $x_2 = 0$. But this cannot happen at a Nash equilibrium.

■

In the above examples, it is assumed that congestion exists in the economic dispatch. But the outcomes can be quite different. Intuitively, when there is fierce competition between the generators (or the consumers) for the capacity of the constrained line (as in Example 2 and 3), a Nash equilibrium may not exist; when the players do not care much about the capacity of the constrained line (as in Example 1), they focus on their own "local" markets, and an efficient Nash equilibrium may exist. However, we have not found a quantitative classification rule to generalize the above observation.

What is the situation in the real world? On the LMP contour map by California ISO, for example, the prices are almost uniform, but differences always exist. This means that congestion always occurs somewhere, pointing to a possibly suboptimal outcome.

VI. Power Network Second Price Mechanism

Since the nodal pricing mechanism does not always induce the desired outcome, we seek alternative mechanisms. We propose the Power Network Second Price (PNSP) mechanism that always induces an efficient Nash equilibrium.

The system operator's problem is the economic dispatch problem (1)-(4). To solve the problem, the system operator needs to know the c_n's and v_m's. However, in a game-theoretic setting, the strategic players may not reveal their true information. Therefore, we need to design a mechanism that gives them incentives to report truthfully.

Although the standard VCG mechanism can apply to this scenario, it requires the type space to be parameterized, since it is impossible for a player to report an arbitrary real-valued function completely. Such a requirement is unrealistic in practice. Therefore, the desired mechanism must ask each player to report an approximation to the function from a finite dimensional bid space. Meanwhile, *dominant strategy implementation* is not expected to be achieved; instead, we seek a *Nash implementation*.

In the proposed mechanism, we adopt the same bid format and the same dispatch rule as in the economic dispatch game. That is, generator n reports (a_n, s_n), and consumer m reports (b_m, d_m). The mechanism then determines the dispatch $z = (x_1, \ldots, x_N, y_1, \ldots, y_M)$ as a solution of the optimization problem (6)-(11). However, the payment rules (and hence the payoff functions) are different, as specified below.

Let $z^{-n,g} = (x_1^{-n,g}, \ldots, x_N^{-n,g}, y_1^{-n,g}, \ldots, y_M^{-n,g})$ be the solution with $s_n = 0$, i.e., when generator n is not present. Generator n is paid

$$w_n^g = \Big(\sum_m b_m y_m - \sum_{n' \neq n} a_{n'} x_{n'} \Big) - \Big(\sum_m b_m y_m^{-n,g} - \sum_{n' \neq n} a_{n'} x_{n'}^{-n,g} \Big), \qquad (12)$$

which is the *positive externality* that generator n imposes on the other players by his participation. Then, his payoff is

$$u_n^g = w_n^g - c_n(x_n).$$

Similarly, denote the solution with $d_m = 0$ by $z^{-m,c} = (x_1^{-m,c}, \ldots, x_N^{-m,c}, y_1^{-m,c}, \ldots, y_M^{-m,c})$, i.e., when consumer m is not present. The payment made by consumer m is

$$w_m^c = \Big(\sum_{m' \neq m} b_{m'} y_{m'}^{-m,c} - \sum_n a_n x_n^{-m,c} \Big) - \Big(\sum_{m' \neq m} b_{m'} y_{m'} - \sum_n a_n x_n \Big), \tag{13}$$

which is the externality that consumer m imposes on the other players by his participation. Then, his payoff is

$$u_m^c = v_m(y_m) - w_n^c.$$

We show that the above mechanism induces an efficient Nash equilibrium.

Theorem 2: There exists an efficient Nash equilibrium in the PNSP mechanism specified by (6)-(13).

Proof: Let $z^* = (x_1^*, \ldots, x_N^*, y_1^*, \ldots, y_M^*)$ be an economic dispatch that solves (1)-(4). Consider the strategy profile: $(a_n, s_n) = (c_n'(x_n^*), x_n^*)$ for all n, $(b_m, d_m) = (v_m'(y_m^*), y_m^*)$ for all m. It is easy to check that z^* also solves (6)-(11) under this strategy profile. It remains to show that the constructed strategy profile is a Nash equilibrium.

Consider generator n'. His current payoff is

$$u_{n'}^g = \Big(\sum_m b_m y_m^* - \sum_{n \neq n'} a_n x_n^* \Big) - \Big(\sum_m b_m y_m^{-n',g} - \sum_{n \neq n'} a_n x_n^{-n',g} \Big) - c_{n'}(x_{n'}^*).$$

Suppose he changes his bid, resulting in a new allocation $\tilde{z} = (\tilde{x}_1, \ldots, \tilde{x}_N, \tilde{y}_1, \ldots, \tilde{y}_M)$. Define the associated notation $\tilde{z}^{-n,g} = (\tilde{x}_1^{-n,g}, \ldots, \tilde{x}_N^{-n,g}, \tilde{y}_1^{-n,g}, \ldots, \tilde{y}_M^{-n,g})$ similarly as before. Then his payoff will be

$$\tilde{u}_{n'}^g = \Big(\sum_m b_m \tilde{y}_m - \sum_{n \neq n'} a_n \tilde{x}_n \Big) - \Big(\sum_m b_m \tilde{y}_m^{-n',g} - \sum_{n \neq n'} a_n \tilde{x}_n^{-n',g} \Big) - c_{n'}(\tilde{x}_{n'}).$$

So his payoff changes by

$$\begin{aligned}
&\tilde{u}_{n'}^g - u_{n'}^g \\
&= \Big(\sum_m b_m \tilde{y}_m - \sum_{n \neq n'} a_n \tilde{x}_n \Big) \\
&\quad - \Big(\sum_m b_m y_m^* - \sum_{n \neq n'} a_n x_n^* \Big) \\
&\quad + c_{n'}(x_{n'}^*) - c_{n'}(\tilde{x}_{n'}) \qquad (14) \\
&= \sum_m b_m (\tilde{y}_m - y_m^*) - \sum_{n \neq n'} a_n (\tilde{x}_n - x_n^*) \\
&\quad + c_{n'}(x_{n'}^*) - c_{n'}(\tilde{x}_{n'}) \\
&= \sum_m v_m'(y_m^*)(\tilde{y}_m - y_m^*) - \sum_{n \neq n'} c_n'(x_n^*)(\tilde{x}_n - x_n^*) \\
&\quad + c_{n'}(x_{n'}^*) - c_{n'}(\tilde{x}_{n'}) \\
&\leq c_{n'}'(x_{n'}^*)(\tilde{x}_{n'} - x_{n'}^*) + c_{n'}(x_{n'}^*) - c_{n'}(\tilde{x}_{n'}) \qquad (15) \\
&\leq 0. \qquad (16)
\end{aligned}$$

Equation (14) follows because $z^{-n,g} = \tilde{z}^{-n,g}$. Since z^* maximizes the concave objective function $S(z) = \sum_m v_m(y_m) - \sum_n c_n(x_n)$ over a convex set P (determined by (2)-(4)), we have

$$\nabla S(z^*) \cdot (z - z^*) \leq 0, \quad \forall z \in P.$$

In particular, letting $z = \tilde{z}$, we get

$$\sum_m v_m'(y_m^*)(\tilde{y}_m - y_m^*) - \sum_n c_n'(x_n^*)(\tilde{x}_n - x_n^*) \leq 0,$$

from which equation (15) follows. Equation (16) follows from the property of convexity.

Thus, he has no incentive to deviate. Similarly, we can show that each consumer m has no incentive to deviate. This proves that the constructed strategy profile is a Nash equilibrium. ∎

VII. Conclusion

We provide a comprehensive framework of studying the strategic interactions under the nodal pricing mechanism. We show by counterexamples that when congestion occurs somewhere, a Nash equilibrium may not exist. We also prove that when there is no congestion, not only a Nash equilibrium but also an efficient one exists. These results suggest that investing in the transmission infrastructure to avoid congestion in the power network can promote the social welfare. As an alternative, the proposed PNSP mechanism can always induce an efficient Nash equilibrium. However, one issue is that budget balance is not achieved in the PNSP mechanism; in fact, it can incur a budget deficit. Moreover, there also exist inefficient Nash equilibria in the PNSP mechanism (which also happens in the nodal pricing mechanism). Thus, new designs of pricing mechanisms are needed to avoid the issues.

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