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Dynamic Economic Dispatch among Strategic Generators with Storage Systems

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***Abstract*—We consider a dynamic economic dispatch problem in wholesale electricity markets. The key feature is that each generator has its own energy storage system, which makes the problem coupled across the time horizon. To implement the optimal dispatch among strategic generators, one major challenge is that the independent system operator is unaware of the operation of the storages. Nevertheless, we show that under certain conditions, the locational marginal pricing mechanism, defined on the static economic dispatch problem, can still induce an efficient Nash equilibrium in the dynamic setting. We also present an alternative pricing mechanism that can induce an efficient Nash equilibrium in a wider range of scenarios.**

I. Introduction

Electric power is traded in wholesale electricity markets involving participants like generators and distributors. The dispatch and pricing of power in a deregulated market is typically coordinated by an independent system operator (ISO). For each scheduling period, the participants submit supply and demand bids to the ISO, who then determines an optimal dispatch that maximizes the social welfare while satisfying certain unit and system constraints. This is known as the static economic dispatch (SED) problem.

Upon computing the economic dispatch, the ISO also dictates the payment to be made or received by each participant. Since the participants are selfish agents, they may not have incentives to reveal their true characteristics. Rather, they play strategically and may even give misleading information if it benefits them. This is not far-fetched; for example, Enron energy traders indeed found ways to manipulate the congestion prices that led to the California electricity crisis of 2000–01 [1], [2]. Therefore, the payment rule, or the pricing mechanism, should be carefully designed to ensure the efficiency of the market and the stability of the underlying power system.

Locational marginal pricing (LMP) is a well-known pricing mechanism that is widely used in the United States, Canada, Australia and New Zealand [3], [4], [5], [6], [7]. The locational marginal price (LMP) at a bus is the cost of supplying the next increment of load at the same bus, taking into account transmission losses and congestion. Since the LMP mechanism dictates a uniform market-clearing price for each bus (or node), it is also called the nodal pricing mechanism.

The LMP mechanism has been studied extensively but primarily in a competitive market setting where the participants are assumed to be truthful. In that case, it is economically efficient, since the LMPs sustain a competitive equilibrium that implements an optimal dispatch. However, exercise of market power is frequently observed in practice [8], which may distort the dispatch.

Starting from our earlier work [9], we have developed a game-theoretic framework to study the strategic interactions under the LMP mechanism. On the one hand, it can be shown that a Nash equilibrium may not exist, and that even when it exists, the price of anarchy may be arbitrarily large. On the other hand, we provide sufficient conditions for the existence of an efficient Nash equilibrium in most practical scenarios. Our work differs from the literature on supply function equilibria [10], [11], [12], [13], since we consider the network topology and adopt piecewise linear bids as used in practice.

One important feature of the future smart grid is the fast development and deployment of energy storage systems [14]. A storage system is considered key to opening the possibility of high penetration of wind and solar generation [15]. Storage systems will greatly enhance the controllability of the grid as well. A lot of recent work has featured the use of storages in various contexts [16], [17], [18].

In this paper, we formulate a dynamic economic dispatch (DED) problem as an extension of the SED problem, by dividing the entire scheduling period into small intervals. For simplicity, we assume that the demand side is competitive, with predicted time-varying load. The generators are strategic agents, with time-varying cost functions that can be due to the ownership of renewable energy generation. The key feature of our model is that each generator has its own storage system, which can be employed to reduce the overall cost. Due to the operation of the storages, the resulting multistage problem is coupled across the time horizon.

Our goal is to design pricing mechanisms that implement the optimal dispatch for the DED problem. Besides the difficulties embedded in the static setting, there is a major challenge arising from the dynamic setting. That is, the ISO may not be aware of the operation of the storages owned by the generators; instead, it still solves independent SED problems over time, while the generators acknowledge this fact. This further complicates the generators' incentives.

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Despite the challenges, we show that under certain conditions, the LMP mechanism, defined on the SED problem, still works in the DED problem. We provide sufficient conditions under which there exists an efficient Nash equilibrium that implements the optimal dispatch in the DED problem. Alternatively, we present the dynamic power network second price (DPNSP) mechanism that can induce an efficient Nash equilibrium in a wider range of scenarios, at the expense that price discrimination may exist.

We note that a variety of DED models have been proposed in the literature, most of which consider ramp rate limits as the time-coupling constraints, and focus on developing efficient algorithms to compute the optimal dispatch [19], [20], [21]. By comparison, we incorporate the network topology and the storages into the model, and our focus is to design efficient pricing mechanisms for strategic generators under the storage-unaware assumption for the ISO.

The paper is organized as follows. First, we introduce the DED problem in Section II. Then we formulate the DED game in Section III. Our main results, the sufficient conditions for the existence of efficient Nash equilibria, are presented in Section IV. We propose the DPNSP mechanism in Section V. In Section VI, we present numerical results to complement the analysis in the previous sections. Section VII concludes the paper.

II. Problem Statement

A. Model

We consider a connected power network which consists of I buses indexed by $i = 1, \ldots, I$, and L branches (i.e., transmission lines) indexed by $l = 1, \ldots, L$. Let $C_l > 0$ be the flow limit of branch l. Define $\boldsymbol{C} = (C_1, \cdots, C_L)$. By convention, such vectors are interpreted as column vectors hereafter.

There are N generators, indexed by $n = 1, \ldots, N$. The number of generators located at each bus can be arbitrary. Let $\boldsymbol{G} = [G_{in}]_{I\times N}$ be the bus-generator incidence matrix, where $G_{in} = 1$ if generator n is located at bus i, and $G_{in} = 0$ otherwise.

Let $\boldsymbol{B} = [B_{ij}]_{I\times I}$ be the bus susceptance matrix, where $B_{ij} \geq 0$ for $i \neq j$ and $B_{ii} = -\sum_{j\neq i} B_{ij}$ for all i. Note that $\boldsymbol{B}$ is the imaginary part of the bus admittance matrix. We also define a matrix $\boldsymbol{H} = [H_{li}]_{L\times I}$, where $H_{li} = B_{ij}$, $H_{lj} = -B_{ij}$, and $H_{lk} = 0$ for $k \neq i, j$, if branch l connects bus i and j. We call $\boldsymbol{H}$ the branch-bus susceptance matrix; note that this is not a standard terminology.

The horizon of interest is typically one day that is discretized into T slots, indexed by $t = 1, \ldots, T$. Let $\theta_{i,t}$ be the voltage phase angle at bus i at time t. Define $\boldsymbol{\theta}_t = (\theta_{1,t}, \ldots, \theta_{I,t})$ and $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_T)$. We assume predicted time-varying load at each bus. Let $D_{i,t} \geq 0$ be the load at bus i at time t, and define $\boldsymbol{D}_t = (D_{1,t}, \ldots, D_{I,t})$.

Each generator has its own storage system. For simplicity, we assume that the storage capacity is sufficiently large. Inclusion of capacity limit, charging rate limit and round-trip efficiency is left for future work. At time t, generator n generates $z_{n,t}$ amount of power, injects $x_{n,t}$ into the grid, and charge the storage by $z_{n,t} - x_{n,t}$ when $z_{n,t} - x_{n,t} > 0$ (or discharge the storage by $x_{n,t} - z_{n,t}$ when $z_{n,t} - x_{n,t} < 0$). Let $y_{n,t}$ be the storage level of generator n at the end of time t, and $y_{n,0} = 0$ be the initial state. Then the storage dynamics is given by

TABLE I
Nomenclature

i	bus index
l	branch index
n	generator index
t	time index
$\boldsymbol{C}$	branch flow limit vector
$\boldsymbol{G}$	bus-generator incidence matrix
$\boldsymbol{B}$	bus susceptance matrix
$\boldsymbol{H}$	branch-bus susceptance matrix
$\theta_{i,t}$	voltage phase angle at bus i at time t
$D_{i,t}$	load at bus i at time t
$x_{n,t}$	injection of generator n at time t
$y_{n,t}$	storage level of generator n at time t
$z_{n,t}$	generation of generator n at time t
$c_{n,t}$	cost function of generator n at time t

$$y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t}, \ \forall t.$$

Each generator may have some renewable energy generation, so that the generation cost can be time-varying. Denote by $c_{n,t}(z_{n,t})$ the cost function of generator n at time t, which is assumed to be convex, increasing and differentiable. Define $\boldsymbol{x}_n = (x_{n,1}, \ldots, x_{n,T})$, $\boldsymbol{X}_t = (x_{1,t}, \ldots, x_{N,t})$, $\boldsymbol{X} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) = (\boldsymbol{X}_1, \ldots, \boldsymbol{X}_T)$, and similarly for $\boldsymbol{Y}$ and $\boldsymbol{Z}$. We summarize the notations in Table I.

For analytical and computational simplicity, we adopt a DC power flow model as a common practice [22]. It is derived via a sequence of approximations:

1) Branch resistances are negligible;
2) Voltage phase angle differences are small;
3) Voltage magnitudes are constant at 1.0 per unit.

Then the power flow from bus i to j at time t is given by $B_{ij}(\theta_{i,t} - \theta_{j,t})$, and the bus power balance equations at time t can be represented in the following matrix form:

$$\boldsymbol{B}\boldsymbol{\theta}_t + \boldsymbol{G}\boldsymbol{X}_t - \boldsymbol{D}_t = \boldsymbol{0}.$$

The branch power flow constraints at time t are given by

$$-\boldsymbol{C} \leq \boldsymbol{H}\boldsymbol{\theta}_t \leq \boldsymbol{C}.$$

By convention, such inequalities are interpreted as element-wise inequalities hereafter.

B. Dynamic Economic Dispatch

The DED problem is to determine an optimal dispatch (i.e., the profile of generation, injection and storage level) that minimizes the total cost while satisfying certain unit and system constraints. At the unit level, for each generator n, the decision variables are $\boldsymbol{x}_n$, $\boldsymbol{y}_n$ and $\boldsymbol{z}_n$, subject to the storage dynamics equations. At the system level, the decision variables are $\boldsymbol{X}$ and $\boldsymbol{\theta}$, subject to the bus power balance equations and the branch power flow constraints. Formally, the DED problem is modeled as a convex program with linear constraints:

$$\begin{aligned}
&\underset{\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z},\boldsymbol{\theta}}{\text{minimize}} && \sum_t \sum_n c_{n,t}(z_{n,t}) && && \text{(1a)}\\
&\text{subject to} && \boldsymbol{B}\boldsymbol{\theta}_t + \boldsymbol{G}\boldsymbol{X}_t - \boldsymbol{D}_t = \boldsymbol{0},\ \forall t, && [\boldsymbol{\pi}_t] && \text{(1b)}\\
& && \boldsymbol{H}\boldsymbol{\theta}_t \le \boldsymbol{C},\ \forall t, && [\boldsymbol{\lambda}_t] && \text{(1c)}\\
& && -\boldsymbol{H}\boldsymbol{\theta}_t \le \boldsymbol{C},\ \forall t, && [\boldsymbol{\mu}_t] && \text{(1d)}\\
& && \boldsymbol{Y}_t = \boldsymbol{Y}_{t-1} + \boldsymbol{Z}_t - \boldsymbol{X}_t,\ \forall t, && [\boldsymbol{\nu}_t] && \text{(1e)}\\
& && \boldsymbol{X}_t, \boldsymbol{Y}_t, \boldsymbol{Z}_t \ge \boldsymbol{0},\ \forall t, && && \text{(1f)}
\end{aligned}$$

where (1b) is the bus power balance equation, (1c) and (1d) are the branch power flow constraints, (1e) is the storage dynamics equation, and (1f) is the feasibility constraint. For simplicity, we leave the inclusion of other time-coupling constraints such as ramp rate limits for future work.

Since $B_{ii} = -\sum_{j \ne i} B_{ij}$ for all i, (1b) is underdetermined with respect to $\boldsymbol{\theta}_t$. Also, in light of (1c) and (1d), only the phase angle differences matter. Thus, for computational purposes, one may choose bus 1 as the slack bus by setting $\theta_{1,t} = 0$ for all t.

Associate the Lagrange multiplier vectors $\boldsymbol{\pi}_t = (\pi_{1,t}, \ldots, \pi_{I,t})$ with (1b), $\boldsymbol{\lambda}_t = (\lambda_{1,t}, \ldots, \lambda_{L,t})$ with (1c), $\boldsymbol{\mu}_t = (\mu_{1,t}, \ldots, \mu_{L,t})$ with (1d), and $\boldsymbol{\nu}_t = (\nu_{1,t}, \ldots, \nu_{N,t})$ with (1e). Define $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_T)$, and similarly for $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$. We assume that the DED problem (1) is always feasible. It is easily seen that the (refined) Slater's condition is automatically satisfied. Therefore, strong duality holds, and the KKT conditions provide necessary and sufficient conditions for optimality [23]:

$$\begin{aligned}
(\boldsymbol{\nu}_t - \boldsymbol{G}^\intercal \boldsymbol{\pi}_t) \circ \boldsymbol{X}_t = \boldsymbol{0},\ \forall t, && \text{(2a)}\\
\boldsymbol{\nu}_t - \boldsymbol{G}^\intercal \boldsymbol{\pi}_t \ge \boldsymbol{0},\ \forall t, && \text{(2b)}\\
(\nu_{n,t+1} - \nu_{n,t}) y_{n,t} = 0,\ \forall n,\ \forall t, && \text{(2c)}\\
\nu_{n,t+1} - \nu_{n,t} \le 0,\ \forall n,\ \forall t, && \text{(2d)}\\
(c'_{n,t}(z_{n,t}) - \nu_{n,t}) z_{n,t} = 0,\ \forall n,\ \forall t, && \text{(2e)}\\
c'_{n,t}(z_{n,t}) - \nu_{n,t} \ge 0,\ \forall n,\ \forall t, && \text{(2f)}\\
\boldsymbol{B}\boldsymbol{\pi}_t - \boldsymbol{H}^\intercal \boldsymbol{\lambda}_t + \boldsymbol{H}^\intercal \boldsymbol{\mu}_t = \boldsymbol{0},\ \forall t, && \text{(2g)}\\
\boldsymbol{B}\boldsymbol{\theta}_t + \boldsymbol{G}\boldsymbol{X}_t - \boldsymbol{D}_t = \boldsymbol{0},\ \forall t, && \text{(2h)}\\
\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C} \le \boldsymbol{0},\ \forall t, && \text{(2i)}\\
-\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C} \le \boldsymbol{0},\ \forall t, && \text{(2j)}\\
\boldsymbol{\lambda}_t \circ (\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C}) = \boldsymbol{0},\ \forall t, && \text{(2k)}\\
\boldsymbol{\mu}_t \circ (-\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C}) = \boldsymbol{0},\ \forall t, && \text{(2l)}\\
\boldsymbol{Y}_t - \boldsymbol{Y}_{t-1} - \boldsymbol{Z}_t + \boldsymbol{X}_t = \boldsymbol{0},\ \forall t, && \text{(2m)}\\
\boldsymbol{X}_t, \boldsymbol{Y}_t, \boldsymbol{Z}_t, \boldsymbol{\lambda}_t, \boldsymbol{\mu}_t \ge \boldsymbol{0},\ \forall t, && \text{(2n)}
\end{aligned}$$

where the symbol $\circ$ denotes the element-wise product of two matrices (in particular, vectors) of the same dimension. For notational cleanliness, we have introduced an additional Lagrange multiplier $\nu_{n,T+1} = 0$ for all n.

III. Game-Theoretic Formulation

The solution to the DED problem (1) gives a socially optimal dispatch, which minimizes the total cost of the system. To solve for the optimal dispatch, the ISO needs to know the true cost functions of the generators. Since the generators are selfish agents, they may not have incentives to reveal their cost functions truthfully. This motivates the game-theoretic formulation of the DED problem.

Our proposed DED game incorporates two important practices:

1) The generators are only allowed to submit their cost functions (or bids) from a restricted function space (or bid format);
2) Since the ISO is unaware of the operation of the storages, it still solves independent SED problems over time.

We will first define the bid format. Then we will specify the DED game, in which the dispatch is done by solving independent SED problems equipped with the LMP mechanism, while the generators acknowledge this fact.

A. Bid Format

Recall that the cost function $c_{n,t}(\cdot)$ is convex, increasing and differentiable. All such functions constitute an infinite-dimensional function space. On the other hand, the bid format is a finite-dimensional function space in practice. The bid $\bar{b}_{n,t}(\cdot)$ is typically a piecewise linear function with increasing slopes. According to the California ISO [24], for instance, "There are 10 bid segments and 11 associated bid points. Each bid point has a generation (MW) and price (PR) value, which are paired together as MW and price coordinates."

In this paper, we adopt a simplified bid format, which is rich enough for our purposes. The bid $b_{n,t}(\cdot)$ is a two-segment piecewise linear function specified by a four-dimensional signal $(p^-_{n,t}, q^-_{n,t}, p^+_{n,t}, q^+_{n,t})$, where $0 \le p^-_{n,t} \le p^+_{n,t}$ and $0 \le q^-_{n,t} \le q^+_{n,t}$:

$$b_{n,t}(z_{n,t}) = \begin{cases} p^-_{n,t} z_{n,t}, & z_{n,t} \in [0, q^-_{n,t}],\\ p^-_{n,t} q^-_{n,t} + p^+_{n,t}(z_{n,t} - q^-_{n,t}), & z_{n,t} \in (q^-_{n,t}, q^+_{n,t}]. \end{cases}$$

The signal $(p^-_{n,t}, q^-_{n,t}, p^+_{n,t}, q^+_{n,t})$ is also referred to as a bid. The true cost function $c_{n,t}(\cdot)$, the bid $\bar{b}_{n,t}(\cdot)$ used in practice, and the simplified bid $b_{n,t}(\cdot)$ used in this paper are illustrated in Fig. 1.

We note that quadratic bids (as used in the supply function equilibrium literature) provide smooth dispatch, revenue and profit curves that facilitate calculus-based analysis, while piecewise linear bids (adopted by most ISOs) do not produce continuously differentiable dispatch, revenue and profit curves, requiring different analysis techniques [25]. This issue, though, has little impact on our analysis.

B. Dynamic Economic Dispatch Game

At the beginning of the horizon, each generator n submits a bid vector $\boldsymbol{b}_n = (b_{n,1}, \ldots, b_{n,T})$ to the ISO. Define the bid profile $\boldsymbol{b} = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_N)$. Also, define $\boldsymbol{b}_{-n} = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_{n-1}, \boldsymbol{b}_{n+1}, \ldots, \boldsymbol{b}_N)$.

Then the ISO solves T independent SED problems. The SED problem for time t is referred to as the t-SED problem.

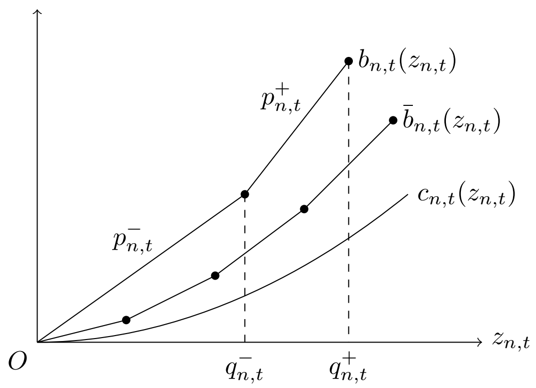

Fig. 1. The true cost function $c_{n,t}(z_{n,t})$, the bid $\bar{b}_{n,t}(z_{n,t})$ used in practice, and the simplified bid $b_{n,t}(z_{n,t})$ specified by a four-dimensional signal $(p^-_{n,t}, q^-_{n,t}, p^+_{n,t}, q^+_{n,t})$.

Given $(b_{1,t}, \ldots, b_{N,t})$, the t-SED problem is the following:

$$\begin{aligned}
\underset{\boldsymbol{X}_t,\boldsymbol{\theta}_t}{\text{minimize}} \quad & \sum_n b_{n,t}(x_{n,t}) && \text{(3a)}\\
\text{subject to} \quad & \boldsymbol{B}\boldsymbol{\theta}_t + \boldsymbol{G}\boldsymbol{X}_t - \boldsymbol{D}_t = \boldsymbol{0}, \quad [\boldsymbol{\pi}_t] && \text{(3b)}\\
& \boldsymbol{H}\boldsymbol{\theta}_t \le \boldsymbol{C}, \quad [\boldsymbol{\lambda}_t] && \text{(3c)}\\
& -\boldsymbol{H}\boldsymbol{\theta}_t \le \boldsymbol{C}, \quad [\boldsymbol{\mu}_t] && \text{(3d)}\\
& \boldsymbol{X}_t \ge \boldsymbol{0}. && \text{(3e)}
\end{aligned}$$

Compared with the DED problem (1), the true cost functions are replaced by the bids, and the storage dynamics equations are discarded.

Let $(\boldsymbol{X}^*_t, \boldsymbol{\theta}^*_t)$ be the primal optimal solution to the t-SED problem (3). Associate the Lagrange multiplier vectors $\boldsymbol{\pi}_t = (\pi_{1,t}, \ldots, \pi_{I,t})$ with (3b), $\boldsymbol{\lambda}_t = (\lambda_{1,t}, \ldots, \lambda_{L,t})$ with (3c), and $\boldsymbol{\mu}_t = (\mu_{1,t}, \ldots, \mu_{L,t})$ with (3d). Let $(\boldsymbol{\pi}^*_t, \boldsymbol{\lambda}^*_t, \boldsymbol{\mu}^*_t)$ be the dual optimal solution. Then the LMP at bus i at time t is given by $\pi^*_{i,t}$, the interpretation of which is the cost of supplying the next increment of load at that bus at that time. For future reference, we list the KKT conditions of the t-SED problem (3) (which are valid when $b_{n,t}(\cdot)$ is differentiable at $x^*_{n,t}$ for all n):

$$\begin{aligned}
(b'_{n,t}(x_{n,t}) - \pi_{i,t})x_{n,t} = 0,\ \forall(n,i) \text{ with } G_{in} = 1, && \text{(4a)}\\
b'_{n,t}(x_{n,t}) - \pi_{i,t} \ge 0,\ \forall(n,i) \text{ with } G_{in} = 1, && \text{(4b)}\\
\boldsymbol{B}\boldsymbol{\pi}_t - \boldsymbol{H}^\intercal\boldsymbol{\lambda}_t + \boldsymbol{H}^\intercal\boldsymbol{\mu}_t = \boldsymbol{0}, && \text{(4c)}\\
\boldsymbol{B}\boldsymbol{\theta}_t + \boldsymbol{G}\boldsymbol{X}_t - \boldsymbol{D}_t = \boldsymbol{0}, && \text{(4d)}\\
\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C} \le \boldsymbol{0}, && \text{(4e)}\\
-\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C} \le \boldsymbol{0}, && \text{(4f)}\\
\boldsymbol{\lambda}_t \circ (\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C}) = \boldsymbol{0}, && \text{(4g)}\\
\boldsymbol{\mu}_t \circ (-\boldsymbol{H}\boldsymbol{\theta}_t - \boldsymbol{C}) = \boldsymbol{0}, && \text{(4h)}\\
\boldsymbol{X}_t, \boldsymbol{\lambda}_t, \boldsymbol{\mu}_t \ge \boldsymbol{0}. && \text{(4i)}
\end{aligned}$$

Once the ISO determines the injection $\boldsymbol{X}^*$ and the LMPs $\boldsymbol{\pi}^*$, each generator independently determines the optimal $\boldsymbol{y}^*_n$ and $\boldsymbol{z}^*_n$ to maximize its own payoff. That is, the payoff function of generator n located at bus i is given by

$$u_n = \sum_t (\pi^*_{i,t} x^*_{n,t} - c_{n,t}(z^*_{n,t})),$$

where $(\boldsymbol{y}^*_n, \boldsymbol{z}^*_n)$ solves the generator's individual problem, referred to as the n-GEN problem:

$$\begin{aligned}
\underset{\boldsymbol{y}_n,\boldsymbol{z}_n}{\text{minimize}} \quad & \sum_t c_{n,t}(z_{n,t}) && \text{(5a)}\\
\text{subject to} \quad & y_{n,t} = y_{n,t-1} + z_{n,t} - x^*_{n,t},\ \forall t, \quad [\nu_{n,t}] && \text{(5b)}\\
& y_{n,t}, z_{n,t} \ge 0,\ \forall t. && \text{(5c)}
\end{aligned}$$

For future reference, we list the KKT conditions of the n-GEN problem (5):

$$\begin{aligned}
(\nu_{n,t+1} - \nu_{n,t})y_{n,t} = 0,\ \forall t, && \text{(6a)}\\
\nu_{n,t+1} - \nu_{n,t} \le 0,\ \forall t, && \text{(6b)}\\
(c'_{n,t}(z_{n,t}) - \nu_{n,t})z_{n,t} = 0,\ \forall t, && \text{(6c)}\\
c'_{n,t}(z_{n,t}) - \nu_{n,t} \ge 0,\ \forall t, && \text{(6d)}\\
y_{n,t} - y_{n,t-1} - z_{n,t} + x^*_{n,t} = 0,\ \forall t, && \text{(6e)}\\
y_{n,t}, z_{n,t} \ge 0,\ \forall t. && \text{(6f)}
\end{aligned}$$

This completes the specification of the DED game. To sum up, the strategy of each generator n is the bid vector $\boldsymbol{b}_n$. The solution to the t-SED problem (3) for all t gives the injection $\boldsymbol{X}^*$ and the LMPs $\boldsymbol{\pi}^*$, which further determines the generation $\boldsymbol{Z}^*$ (by solving the n-GEN problem (5) for all n) and thus the payoff u_n for all n.

We adopt the (pure) Nash equilibrium as the solution concept of the DED game. A bid profile $\boldsymbol{b}$ is a Nash equilibrium if no generator can be better off by deviating unilaterally. A Nash equilibrium $\boldsymbol{b}$ is efficient if $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*)$, the solution to the DED game (3) and (5) given $\boldsymbol{b}$, is also a solution to the DED problem (1).

IV. Existence of Efficient Nash Equilibria

We are interested in whether an efficient Nash equilibrium exists in the DED game. Under certain conditions, the answer is yes. The idea is the following. First, when there is enough competition, each generator can be incentivized to submit a "truthful" bid that is an adequate approximation of its true cost function. Second, corresponding to the true cost functions in the DED problem (1), there is a dispatch-equivalent bid profile in the DED game (3) and (5). In other words, while the ISO solves the SED problems, the generators acknowledge this fact and play accordingly, so that the optimal dispatch for the DED problem may still be achieved.

We will present two sufficient conditions under either of which there exist an efficient Nash equilibrium. In fact, it is even possible to compute such equilibria directly, as shown by the constructive proofs.

Unless otherwise specified, we make the following assumption which ensures that no generator has enough market power to ask for arbitrarily high prices.

Assumption 1: The DED problem (1) is still feasible if any one of the generators is excluded.

A. Congestion-Free Condition

Definition 1 (Congestion-Free Condition): A DED game satisfies the congestion-free condition, if no branch power flow constraint (1c) or (1d) is binding in the DED problem (1).

The following lemma shows the uniformity of LMPs at each time in the DED problem under this condition.

Lemma 1: Under the congestion-free condition, all the LMPs are equal at each time in the DED problem.

Proof: Fix a time t. Under the congestion-free condition, $\boldsymbol{\lambda}_t = \boldsymbol{\mu}_t = \mathbf{0}$. From the KKT condition (2g), we have $\boldsymbol{B}\boldsymbol{\pi}_t = \mathbf{0}$. Let $\mathcal{I}_t = \arg\max_i \pi_{i,t}$ be the set of buses with the largest LMPs. Then for each $i \in \mathcal{I}_t$,

$$\sum_j B_{ij}\pi_{j,t} = 0,$$

or

$$\sum_{j\neq i} B_{ij}(\pi_{j,t} - \pi_{i,t}) = 0.$$

Since $\pi_{j,t} \le \pi_{i,t}$ and $B_{ij} \ge 0$ for $j \ne i$, we must have $\pi_{j,t} = \pi_{i,t}$ for j connected to i (i.e., $B_{ij} > 0$). It follows from the connectedness of the network that all the buses belong to $\mathcal{I}_t$. That is, all the LMPs are equal. ■

It is immediate to prove by construction the existence of efficient Nash equilibria in the DED game, under the congestion-free condition that is defined on the DED problem. In the following, let Q be a large enough constant, say the aggregate load over the horizon, $\sum_t \sum_i D_{i,t}$.

Theorem 1: Under Assumption 1 and the congestion-free condition, there exists an efficient Nash equilibrium in the DED game.

Proof: Let $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*)$ be a primal optimal solution to the DED problem (1), and $(\boldsymbol{\pi}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ be a dual optimal solution. Then $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*, \boldsymbol{\pi}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ satisfies the KKT conditions (2). By Lemma 1, $\pi^*_{i,t} = \pi^*_{1,t}$ for all i and for all t.

Let $\boldsymbol{b}$ be a bid profile where $p^-_{n,t} = p^+_{n,t} = \pi^*_{1,t}$, $q^-_{n,t} = x^*_{n,t}$, $q^+_{n,t} = Q$, for all n and for all t. Given the bid profile $\boldsymbol{b}$, it is easy to check that $(\boldsymbol{X}^*_t, \boldsymbol{\theta}^*_t, \boldsymbol{\pi}^*, \boldsymbol{\lambda}^*_t, \boldsymbol{\mu}^*_t)$ satisfies the KKT conditions (4), so that $(\boldsymbol{X}^*_t, \boldsymbol{\theta}^*_t)$ is a solution to the t-SED problem (3) for all t. Similarly, since $(\boldsymbol{y}^*_n, \boldsymbol{z}^*_n, \nu^*_{n,t}, \forall t)$ satisfies the KKT conditions (6), $(\boldsymbol{y}^*_n, \boldsymbol{z}^*_n)$ is a solution to the n-GEN problem (5) for all n. That is, the bid profile $\boldsymbol{b}$ indeed induces the optimal dispatch. It remains to show that $\boldsymbol{b}$ is a Nash equilibrium.

Consider generator n located at bus i. From the KKT conditions, it can be easily seen that $(\boldsymbol{x}^*_n, \boldsymbol{y}^*_n, \boldsymbol{z}^*_n)$ is also a solution to the following problem:

$$\begin{aligned}\underset{\boldsymbol{x}_n, \boldsymbol{y}_n, \boldsymbol{z}_n}{\text{maximize}} \quad & \sum_t (\pi^*_{1,t} x_{n,t} - c_{n,t}(z_{n,t})) \\ \text{subject to} \quad & y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t},\ \forall t, \\ & x_{n,t}, y_{n,t}, z_{n,t} \ge 0,\ \forall t.\end{aligned}$$

Its current payoff is

$$u_n = \sum_t (\pi^*_{1,t} x^*_{n,t} - c_{n,t}(z^*_{n,t})).$$

Suppose it changes its bid to $\hat{\boldsymbol{b}}_n$. Let $(\hat{\boldsymbol{X}}, \hat{\boldsymbol{Y}}, \hat{\boldsymbol{Z}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\nu}})$ be the solution to the DED game (3) and (5) given $(\hat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n})$. By Assumption 1 and in light of $\boldsymbol{b}_{-n}$ (where $b_{m,t}(x_{m,t}) = \pi^*_{1,t} x_{m,t}$ with a large enough cap Q for $m \ne n$), we have $\hat{\pi}_{i,t} \le \pi^*_{1,t}$ for all t. Its payoff will be

$$\begin{aligned}\hat{u}_n &= \sum_t (\hat{\pi}_{i,t}\hat{x}_{n,t} - c_{n,t}(\hat{z}_{n,t})) \\ &\le \sum_t (\pi^*_{1,t}\hat{x}_{n,t} - c_{n,t}(\hat{z}_{n,t})) \\ &\le \sum_t (\pi^*_{1,t} x^*_{n,t} - c_{n,t}(z^*_{n,t})) \\ &= u_n.\end{aligned}$$

Thus, it has no incentive to deviate. This proves that the constructed bid profile is a Nash equilibrium. ■

We note that each $b_{n,t}(\cdot)$ is linear (with a large enough cap Q) in the constructed bid profile. Therefore, the value of $q^-_{n,t}$ does not matter, but it provides a "suggested" dispatch point for the ISO for tie-breaking purposes. To put it another way, if we let $p^+_{n,t} = \pi^*_{1,t} + \delta$ with a small enough $\delta > 0$ for all n and for all t in $\boldsymbol{b}$, then we obtain an efficient ϵ-Nash equilibrium (which induces the same dispatch as $\boldsymbol{b}$ does).

B. Monopoly-Free Condition

Definition 2 (Monopoly-Free Condition): A DED game satisfies the monopoly-free condition, if there are at least two generators at each bus.

This condition is easier to use than the congestion-free condition, since we only need to know the placement of the generators. Moreover, Assumption 1 is automatically satisfied under this condition. The proof of the existence of efficient Nash equilibria is similar as before.

Theorem 2: Under the monopoly-free condition, there exists an efficient Nash equilibrium in the DED game.

Proof: Let $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*)$ be a primal optimal solution to the DED problem (1), and $(\boldsymbol{\pi}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ be a dual optimal solution. Then $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*, \boldsymbol{\pi}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ satisfies the KKT conditions (2).

Let $\boldsymbol{b}$ be a bid profile where $p^-_{n,t} = p^+_{n,t} = \pi^*_{i,t}$, $q^-_{n,t} = x^*_{n,t}$, $q^+_{n,t} = Q$, for all (n,i) with $G_{in} = 1$ and for all t. Given the bid profile $\boldsymbol{b}$, it is easy to check that $(\boldsymbol{X}^*_t, \boldsymbol{\theta}^*_t, \boldsymbol{\pi}^*_t, \boldsymbol{\lambda}^*_t, \boldsymbol{\mu}^*_t)$ satisfies the KKT conditions (4), so that $(\boldsymbol{X}^*_t, \boldsymbol{\theta}^*_t)$ is a solution to the t-SED problem (3) for all t. Similarly, since $(\boldsymbol{y}^*_n, \boldsymbol{z}^*_n, \nu^*_{n,t}, \forall t)$ satisfies the KKT conditions (6), $(\boldsymbol{y}^*_n, \boldsymbol{z}^*_n)$ is a solution to the n-GEN problem (5) for all n. That is, the bid profile $\boldsymbol{b}$ indeed induces the optimal dispatch. It remains to show that $\boldsymbol{b}$ is a Nash equilibrium.

Consider generator n located at bus i. From the KKT conditions, it can be easily seen that $(\boldsymbol{x}^*_n, \boldsymbol{y}^*_n, \boldsymbol{z}^*_n)$ is also a solution to the following problem:

$$\begin{aligned}\underset{\boldsymbol{x}_n, \boldsymbol{y}_n, \boldsymbol{z}_n}{\text{maximize}} \quad & \sum_t (\pi^*_{i,t} x_{n,t} - c_{n,t}(z_{n,t})) \\ \text{subject to} \quad & y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t},\ \forall t, \\ & x_{n,t}, y_{n,t}, z_{n,t} \ge 0,\ \forall t.\end{aligned}$$

Its current payoff is

$$u_n = \sum_t (\pi_{i,t}^* x_{n,t}^* - c_{n,t}(z_{n,t}^*)).$$

Suppose it changes its bid to $\hat{\boldsymbol{b}}_n$. Let $(\hat{\boldsymbol{X}}, \hat{\boldsymbol{Y}}, \hat{\boldsymbol{Z}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\nu}})$ be the solution to the DED game (3) and (5) given $(\hat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n})$. Under the monopoly-free condition, there is at least another generator m located at the same bus i, whose bid is $b_{m,t}(x_{m,t}) = \pi_{i,t}^* x_{m,t}$ with a large enough cap Q. So we have $\hat{\pi}_{i,t} \leq \pi_{i,t}^*$ for all t. Its payoff will be

$$\begin{aligned} \hat{u}_n &= \sum_t (\hat{\pi}_{i,t} \hat{x}_{n,t} - c_{n,t}(\hat{z}_{n,t})) \\ &\leq \sum_t (\pi_{i,t}^* \hat{x}_{n,t} - c_{n,t}(\hat{z}_{n,t})) \\ &\leq \sum_t (\pi_{i,t}^* x_{n,t}^* - c_{n,t}(z_{n,t}^*)) \\ &= u_n. \end{aligned}$$

Thus, it has no incentive to deviate. This proves that the constructed bid profile is a Nash equilibrium. ■

Note that neither of these conditions is necessary for the existence of (even non-efficient) Nash equilibria. On the other hand, they may be adapted for a larger class of scenarios. We leave the extensions for future work.

V. Dynamic Power Network Second Price Mechanism

In the mechanism design theory, the Vickrey-Clarke-Groves (VCG) mechanism is a canonical mechanism that implements efficient allocations in dominant strategies [26]. However, the standard VCG mechanism does not apply directly in our context, since we require a finite-dimensional (and preferably low-dimensional) bid format while the space of true cost functions is infinite-dimensional. In this section, we adapt the standard VCG mechanism and propose a VCG-type pricing mechanism, the DPNSP mechanism. This alternative mechanism can induce an efficient Nash equilibrium in a wider range of scenarios than the LMP mechanism.

Note that we do not change the bid format or the dispatch rule (3) and (5); in particular, the storage-unaware assumption for the ISO still holds. We only change the payment rule, so that the payoff function of generator n is now given by

$$u_n = w_n - \sum_t c_{n,t}(z_{n,t}^*),$$

where w_n is the payment made to generator n, as specified below.

Given a bid profile $\boldsymbol{b}$, let $(\boldsymbol{X}_t^*, \boldsymbol{\theta}_t^*)$ be the solution to the t-SED problem (3) for all t. Also, let $(\boldsymbol{X}_t^{-n}, \boldsymbol{\theta}_t^{-n})$ be the solution to the t-SED problem (3) for all t when generator n is excluded, so that $x_{n,t}^{-n} = 0$ for all t. Then w_n is given by

$$\begin{aligned} w_n &= \sum_t \sum_{m \neq n} b_{m,t}(x_{m,t}^{-n}) - \sum_t \sum_{m \neq n} b_{m,t}(x_{m,t}^*) \\ &= \sum_t \sum_{m \neq n} (b_{m,t}(x_{m,t}^{-n}) - b_{m,t}(x_{m,t}^*)), \end{aligned}$$

which is the positive externality that generator n imposes on the other agents by its participation.

We show that the DPNSP mechanism always induces an efficient Nash equilibrium under a very mild condition.

Theorem 3: Under Assumption 1, there exists an efficient Nash equilibrium in the DED game equipped with the DPNSP mechanism.

Proof: Let $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*)$ be a primal optimal solution to the DED problem (1), and $(\boldsymbol{\pi}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ be a dual optimal solution. Then $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{\theta}^*, \boldsymbol{\pi}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ satisfies the KKT conditions (2).

Let $\boldsymbol{b}$ be a bid profile where $p_{n,t}^- = p_{n,t}^+ = \pi_{i,t}^*$, $q_{n,t}^- = x_{n,t}^*$, $q_{n,t}^+ = Q$, for all (n,i) with $G_{in} = 1$ and for all t. Given the bid profile $\boldsymbol{b}$, it is easy to check that $(\boldsymbol{X}_t^*, \boldsymbol{\theta}_t^*, \boldsymbol{\pi}_t^*, \boldsymbol{\lambda}_t^*, \boldsymbol{\mu}_t^*)$ satisfies the KKT conditions (4), so that $(\boldsymbol{X}_t^*, \boldsymbol{\theta}_t^*)$ is a solution to the t-SED problem (3) for all t. Similarly, since $(\boldsymbol{y}_n^*, \boldsymbol{z}_n^*, \nu_{n,t}^*, \forall t)$ satisfies the KKT conditions (6), $(\boldsymbol{y}_n^*, \boldsymbol{z}_n^*)$ is a solution to the n-GEN problem (5) for all n. That is, the bid profile $\boldsymbol{b}$ indeed induces the optimal dispatch. It remains to show that $\boldsymbol{b}$ is a Nash equilibrium.

Consider generator n located at bus i. Its current payoff is

$$u_n = \sum_t \sum_{m \neq n} (b_{m,t}(x_{m,t}^{-n}) - b_{m,t}(x_{m,t}^*)) - \sum_t c_{n,t}(z_{n,t}^*).$$

Note that Assumption 1 ensures that $(\boldsymbol{X}_t^{-n}, \boldsymbol{\theta}_t^{-n})$ exists given $\boldsymbol{b}$. Suppose it changes its bid to $\hat{\boldsymbol{b}}_n$. Let $(\hat{\boldsymbol{X}}, \hat{\boldsymbol{Y}}, \hat{\boldsymbol{Z}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\nu}})$ be the solution to the dispatch rule (3) and (5) given $(\hat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n})$. Define $\hat{\boldsymbol{X}}^{-n}$ similarly as before. Its payoff will be

$$\hat{u}_n = \sum_t \sum_{m \neq n} (b_{m,t}(\hat{x}_{m,t}^{-n}) - b_{m,t}(\hat{x}_{m,t})) - \sum_t c_{n,t}(\hat{z}_{n,t}).$$

So its payoff changes by

$$\begin{aligned} & \hat{u}_n - u_n \\ = & \sum_t \sum_{m \neq n} (b_{m,t}(x_{m,t}^*) - b_{m,t}(\hat{x}_{m,t})) \\ & + \sum_t (c_{n,t}(z_{n,t}^*) - c_{n,t}(\hat{z}_{n,t})) \\ \leq & \sum_t (b_{n,t}(\hat{x}_{n,t}) - b_{n,t}(x_{n,t}^*)) + \sum_t (c_{n,t}(z_{n,t}^*) - c_{n,t}(\hat{z}_{n,t})) \\ = & \sum_t \pi_{i,t}^* (\hat{x}_{n,t} - x_{n,t}^*) + \sum_t (c_{n,t}(z_{n,t}^*) - c_{n,t}(\hat{z}_{n,t})) \\ \leq & \ 0. \end{aligned}$$

The first equality follows from the fact that $\hat{x}_{m,t}^{-n} = x_{m,t}^{-n}$ for $m \neq n$ and for all t. Since $(\boldsymbol{X}_t^*, \boldsymbol{\theta}_t^*)$ is a solution to the t-SED problem (3) for all t, we have

$$\sum_m b_{m,t}(x_{m,t}^*) \leq \sum_m b_{m,t}(\hat{x}_{m,t}), \ \forall t,$$

so that the first inequality follows. From the KKT conditions, it can be easily seen that $(\boldsymbol{x}_n^*, \boldsymbol{y}_n^*, \boldsymbol{z}_n^*)$ is also a solution to the following problem:

$$\begin{aligned} \underset{\boldsymbol{x}_n,\boldsymbol{y}_n,\boldsymbol{z}_n}{\text{maximize}} \quad & \sum_t (\pi^*_{i,t} x_{n,t} - c_{n,t}(z_{n,t})) \\ \text{subject to} \quad & y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t}, \ \forall t, \\ & x_{n,t}, y_{n,t}, z_{n,t} \geq 0, \ \forall t, \end{aligned}$$

so that the last inequality follows. Thus, it has no incentive to deviate. This proves that the constructed bid profile is a Nash equilibrium. ∎

We note that like the LMP mechanism, there may also be undesirable Nash equilibria in the DPNSP mechanism. Moreover, the DPNSP mechanism assigns a total payment w_n to each generator n, while the LMP mechanism assigns a uniform price per unit at each bus at each time. That is, the DPNSP mechanism is more applicable, at the expense that price discrimination may exist.

VI. Numerical Results

In this section, we present numerical results for the IEEE power system test cases [27]. On top of the model used in the previous analysis, we impose the same storage capacity limit on each generator. When the limit is zero, the DED problem is equivalent to T independent SED problems.

Our main focus is to show that the DED approach can reduce the total cost by utilizing the storages. We also want to examine the performance under different storage capacity limits. Once the DED problem (1) is solved, we can easily construct the efficient Nash equilibria of the DED game (3) and (5) equipped with either the LMP or the DPNSP mechanism, based on the previous analysis.

We first consider the IEEE 57-bus system. There are 57 buses, 7 generators and 42 loads in this power network. Note that the IEEE test cases do not have branch flow limits, which means the congestion-free condition is automatically satisfied. Inclusion of branch flow limits is left for future work.

Consider a one-day horizon and let $T = 24$. Since the test case only gives the load data for a single scheduling interval, we take the load D_i at bus i as the mean value and model the dynamic load as a negative sine wave:

$$D_{i,t} = D_i[1 - 0.5\sin(\pi t/12)], \ t = 1, 2, \ldots, 24,$$

which mimics the load in real-life scenarios. For the generation cost functions, we use the data from MATPOWER [28], which is listed in Table II. For simplicity, we assume that the cost function of each generator is constant over time.

Fig. 2 shows the aggregate generation versus time under different storage capacity limits. As the storage capacity limit increases, the generation curve becomes flatter, since the cost functions are assumed to be constant over time. In the case where the storage capacity limit is zero, the aggregate generation curve is identical to the aggregate load curve.

Since there is no congestion in the power network, the LMPs are equal at each time. Fig. 3 shows the LMP versus time under different storage capacity limits. As the storage capacity limit increases, the LMP curve becomes flatter. That is, the use of storage can reduce the price volatility.

TABLE II
Generation Cost Functions for the IEEE 57-bus System

Generator	Bus	Cost (\$/hr) (x: MW)
1	1	$0.0776x^2 + 20x$
2	2	$0.0100x^2 + 40x$
3	3	$0.2500x^2 + 20x$
4	6	$0.0100x^2 + 40x$
5	8	$0.0222x^2 + 20x$
6	9	$0.0100x^2 + 40x$
7	12	$0.0323x^2 + 20x$

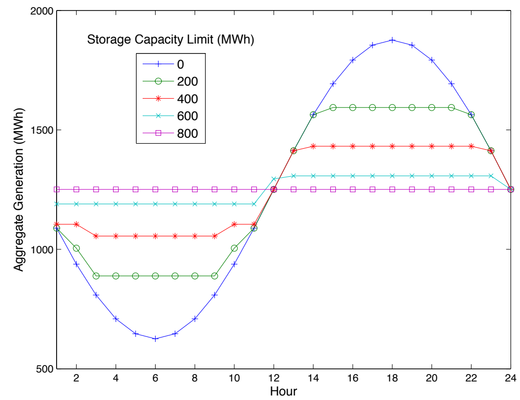

Fig. 2. The aggregate generation versus time under different storage capacity limits in the IEEE 57-bus system.

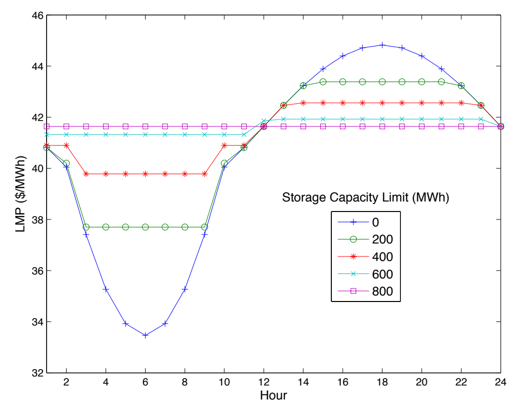

Fig. 3. The LMP versus time under different storage capacity limits in the IEEE 57-bus system.

Then we compare the total cost over the 24-hour horizon among different storage capacity limits, as shown in Fig. 4. As the storage capacity limit increases, the total cost decreases. This shows that the DED approach can reduce the total cost by utilizing the storages to smooth the generation curve. Again, although the ISO still solves the SED problems in the DED game, the generators acknowledge this fact and bid accordingly, so that the optimal solution to the DED problem can be achieved.

Lastly, we compare the total costs between the SED ap-

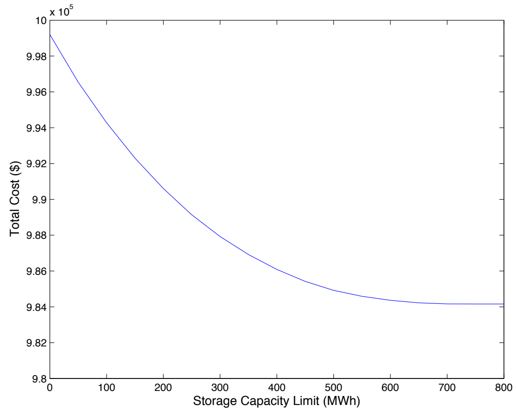

Fig. 4. The total cost versus storage capacity limit in the IEEE 57-bus system.

TABLE III
TOTAL COSTS BETWEEN SED AND DED

Case	SED ($)	DED ($)
14-bus	1.882×10^5	1.834×10^5
30-bus	1.390×10^4	1.357×10^4
57-bus	9.992×10^5	9.842×10^5
118-bus	3.100×10^6	3.023×10^6

proach and the DED approach under different test cases, when the storage capacities are unlimited. From the results listed in Table III, we can see that overall, the DED approach reduces the total cost.

VII. CONCLUSION

We consider a DED problem as an extension of the SED problem. The key feature is that each generator has its own energy storage system, so that the total cost can be reduced. Since the generators are selfish agents, we consider the strategic setting and formulate the DED game, in which the bid format is restricted, and the ISO is unaware of the operation of the storages. Despite the challenges, we provide two sufficient conditions under either of which there exist an efficient Nash equilibrium, which can be computed directly based on the constructive proofs. This tells us that the current market architecture does not need to be changed. Alternatively, we propose the DPNSP mechanism which can induce an efficient Nash equilibrium under a very mild assumption, but may introduce price discrimination.

Due to space constraints, we use a simple model to provide the insights into the DED problem. Some immediate extensions include: additional time-coupling constraints such as ramp rate limits, a more detailed model of the storage system, a double-sided setting involving distributors as strategic agents, etc. In future work, we will investigate whether it is possible to adapt the two sufficient conditions for a larger class of scenarios, and generalize the results to the stochastic setting.

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