Buying Random yet Correlated Wind Power

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Abstract—**We consider an auction design problem, where an aggregator procures wind power from multiple wind farms. While the realized generation of each wind farm is random, the probability distribution can be learned beforehand as its private information. Since the wind farms are geographically close, the distributions are possibly correlated. We formulate a unified optimization problem to study both the welfare-maximizing and the revenue-maximizing objectives. We show that the aggregator may extract the full surplus by exploiting the correlation among the distributions. We also illustrate, through a numerical example, the case where full surplus extraction is not achievable.**

I. Introduction

With the increasing penetration of wind power, challenges arise in integrating such random generation into the current electricity grid and market. While wind power generators are commonly treated as negative loads and receive feed-in tariffs, this scheme would be no longer applicable when the penetration level is high, with substantial reserve margin needed.

Alternatively, wind power generators can be required to participate in the competitive electricity pool through a two-settlement market system. In the ex ante day-ahead (DA) forward market, the wind power generator commits to a fixed amount of generation; in the ex post real-time (RT) spot market, it pays a penalty for the shortfall. The problem of optimizing the contracted amount can be studied as a decision-making problem [1]. In [2], they introduce risky power contracts in addition to firm power contracts to enable flexible and efficient wind power aggregation. They focus on how optimal offerings and equilibria depend on exogenous price signals, and on deriving concept and expressions for critical prices from the perspective of wind power generators.

To address the risk of not meeting operating constraints such as power balance due to the uncertainty of wind power, a new paradigm for power system operation called risk-limiting dispatch (RLD) has been proposed [3]. The RLD employs real-time information about supply and demand, taking into account the stochastic nature of wind power generation, and determines a risk-constrained stochastic optimal dispatch.

The major issue of the aforementioned work is that such implementations require accurate stochastic models for wind power generation, which are possibly private information of wind power generators. Without proper market mechanisms, they may not have incentives to reveal their private information truthfully. This underlies the motivation of our work.

In our model, we consider an aggregator who procures wind power from multiple wind farms through auctions [4], [5]. For simplicity, we assume that only one wind farm is selected as the provider, for technical or regulatory reasons; the others can still resort to outside opportunities since the selection decision is made in the DA market. The wind farms have no marginal cost, and the realized generation of the selected provider has a value to the aggregator. While the realized generation of each wind farm is random, the probability distribution can be learned beforehand as its private information. This motivates the proposed auction paradigm called the stochastic resource auction. The aggregator's objective can be either welfare-maximizing or revenue-maximizing. Our general task is to design incentive compatible auction mechanisms to accommodate various objectives of the aggregator.

To this end, we formulate a unified optimization problem in the mechanism design framework. For the welfare-maximizing objective, there exists a dominant strategy implementation with ex post individual rationality, based on our earlier work [6]. For the revenue-maximizing objective, the solution is commonly called the optimal auction. In the seminal work on optimal auction design [7], a primal assumption is that the agents' types are independent, which, however, may not be the case in our context. Since the wind farms are geographically close, the distributions are possibly correlated: conditioned on the accurate forecast of the local weather, each wind farm also has some coarse estimation of the others', whereas the aggregator only has a prior joint distribution of the types. We adapt the work on correlated mechanism design [8], [9], [10], [11], [12] for the stochastic resource auction. We show that the aggregator may extract the full surplus by exploiting the correlation among the distributions. For theoretical contribution, we propose corollaries for special cases, and establish impossibility results when the constraints are strengthened. Then we provide a numerical example to illustrate the case where full surplus extraction is not achievable.

We note that following the same motivation, one can approach the same problem from a contract design point of view that typically considers a principal and a single agent [13], [14]. In [15], they consider an aggregator procuring both conventional and renewable power from a single producer that has multi-dimensional private information, and show that the

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optimal mechanism is a menu of contracts. As another context, [16] proposes a dynamic contract design problem with an application to indirect load control. The key characteristic of their setting is that the principal has no capability to monitor the agent's control or the state of the engineered system, whereas the agent has perfect observations.

The paper is organized as follows. In Section II, we propose the stochastic resource auction paradigm, and formulate the auction design problem as a unified optimization problem. The results for the welfare-maximizing problem are presented. In Section III, we study the revenue-maximizing problem with correlated types, where we define full correlation that induces full surplus extraction. In Section IV, we specify the type space and provide a numerical example to illustrate both cases, whether the full correlation condition is satisfied or not. Section V concludes the paper.

II. Problem Statement

A. Model

Consider an aggregator who wants to buy wind power (to be delivered at a given future time) from I wind farms (or simply agents), indexed by $i = 1, \dots, I$. Agent i's wind power generation is a random variable X_i, the distribution of which can be parameterized by θ_i. While the realization of X_i cannot be known a priori, agent i learns θ_i (and hence the distribution of X_i) by forecasting the local weather. The parameter θ_i, referred to as agent i's type, is his private information. We assume that θ_i can take finitely many values such that $\theta_i \in \Theta_i = \{\theta_i^1, \dots, \theta_i^{m_i}\}$ for some m_i, where Θ_i is referred to as agent i's type space. Let Θ_{-i} be $\times_{j \neq i} \Theta_j$. The number of elements in Θ_{-i} is denoted by $n_i = \prod_{j \neq i} m_j$. Let $\theta = (\theta_1, \dots, \theta_I)$, $\theta_{-i} = (\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_I)$, and similarly for the other vectors.

While θ_i is agent i's private information, θ is drawn from a commonly known prior distribution $\phi(\cdot)$, which is treated as a probability mass function in this model. Moreover, $\theta_1, \dots, \theta_I$ are not necessarily independent. In fact, the main part of this paper is to exploit the correlation among the agents' types so as to extract the full surplus for the aggregator's sake.

The aggregator, either welfare-maximizing or revenue-maximizing, procures wind power from the agents through an auction. The agents have no marginal cost, and a realized generation X_i has a value of $v(X_i)$ to the aggregator, where $v : [0, \infty) \to \mathbb{R}_+$ is called the valuation function. Since the wind power generation is random, we propose a new auction paradigm, referred to as the stochastic resource auction, as shown in Fig. 1. In the DA market, nature draws θ according to the joint distribution $\phi(\cdot)$. Each agent i learns his own type θ_i and updates his belief in the distribution of the others' types, to the conditional distribution $\phi_i(\theta_{-i}|\theta_i)$. Then each agent i submits a bid $\hat{\theta}_i$ as his reported type, which could be different from the true type θ_i, to the aggregator. Based on the bid profile $\hat{\theta}$, each agent i is selected as the wind power provider with probability $p_i(\hat{\theta})$ and makes a payment $t_i(\hat{\theta})$ to the aggregator. The unselected agents leave the auction (and possibly resort to outside opportunities for profit, since they

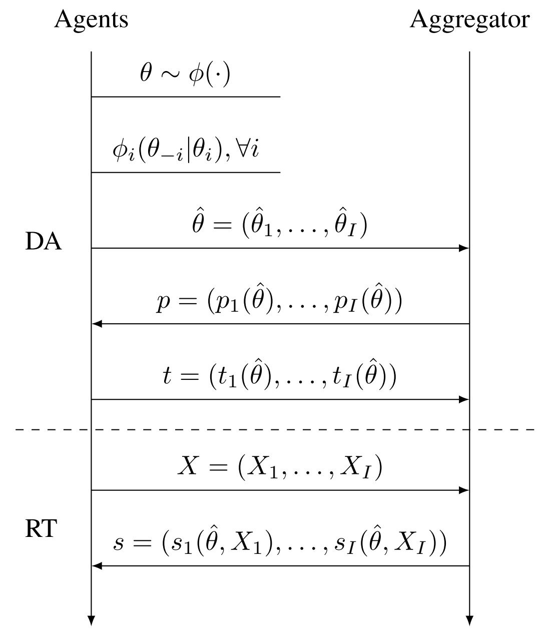

Fig. 1. The stochastic resource auction.

are still in the ex ante stage). In the RT market, upon the realization of X_i, the selected provider i gets paid an amount of $s_i(\hat{\theta}, X_i)$ from the aggregator. It is natural to prescribe $s_i(\hat{\theta}, X_i) = v(X_i)$. Thus, the payoff of agent i is

$$u_i(\hat{\theta}, X_i) = p_i(\hat{\theta}) v(X_i) - t_i(\hat{\theta}).$$

In the stochastic resource auction, the selection and payment schemes define a direct revelation mechanism $\Gamma = \{p, t\}$. We can focus on direct and truthful mechanisms without loss of generality by the revelation principle. While the mechanism is theoretically static, the proposed paradigm is aligned with the existing two-settlement market system. Moreover, when the ex post payment $s_i(\hat{\theta}, X_i)$ is relaxed, we may derive a richer class of mechanisms with proper practical interpretations.

B. Welfare and Revenue Maximization

In the context of auction design, there are mainly two classes of mechanisms: welfare-maximizing mechanisms and revenue-maximizing mechanisms. We first formulate the generic auction design problem, which is then specified for both scenarios.

1) The Generic Problem: The generic auction design problem is the following optimization problem:

$$\begin{aligned} \underset{p,t}{\text{maximize}} \quad & \mathbb{E}[g(\theta, p, t)] && \text{(1a)} \\ \text{subject to} \quad & \text{(IC)}, && \text{(1b)} \\ & \text{(IR)}, && \text{(1c)} \\ & \textstyle\sum_i p_i(\theta) \le 1, \ \forall \theta, && \text{(1d)} \\ & p_i(\theta) \ge 0, \ \forall i, \ \forall \theta, && \text{(1e)} \end{aligned}$$

where the function g represents the aggregator's objective, which we call the objective function. The goal is to choose

$\{p, t\}$ to maximize the expectation of g over the joint distribution of θ, subject to the incentive compatibility (IC) constraint (1b), the individual rationality (IR) constraint (1c), and the feasibility constraints (1d) and (1e).

We consider two notions of incentive compatibility: dominant strategy incentive compatibility (DSIC) and Bayesian incentive compatibility (BIC). The DSIC constraint states that truth telling is each agent i's best strategy for all θ_{-i}:

$$\mathbb{E}[u_i(\theta, X_i)|\theta] \geq \mathbb{E}[u_i(\hat{\theta}_i, \theta_{-i}, X_i)|\theta],\ \forall i,\ \forall\theta,\ \forall\hat{\theta}_i. \quad (2)$$

The BIC constraint is weaker, which states that truth telling is each agent i's best strategy averaging over all θ_{-i}:

$$\mathbb{E}[u_i(\theta, X_i)|\theta_i] \geq \mathbb{E}[u_i(\hat{\theta}_i, \theta_{-i}, X_i)|\theta_i],\ \forall i,\ \forall\theta_i,\ \forall\hat{\theta}_i. \quad (3)$$

There are three notions of individual rationality, corresponding to the three stages at which voluntary participation may be relevant. The ex post IR constraint states that each agent has no incentive to withdraw from the auction after the others' types are revealed:

$$\mathbb{E}[u_i(\theta, X_i)|\theta] \geq 0,\ \forall i,\ \forall\theta. \quad (4)$$

The interim IR constraint is weaker, which states that each agent has no incentive to withdraw from the auction after he learns his own type but before the others' types are revealed:

$$\mathbb{E}[u_i(\theta, X_i)|\theta_i] \geq 0,\ \forall i,\ \forall\theta_i. \quad (5)$$

The ex ante IR constraint is the weakest, which states that each agent has no incentive to withdraw from the auction before he learns his own type:

$$\mathbb{E}[u_i(\theta, X_i)] \geq 0,\ \forall i. \quad (6)$$

2) The Welfare-Maximizing Problem: We define the social welfare as the expected value derived from the generation of the selected provider. Thus, we have the following objective function:

$$g(\theta, p, t) = \mathbb{E}[\textstyle\sum_i p_i(\theta)v(X_i)] = \sum_i p_i(\theta)\bar{v}(\theta_i),$$

where we define $\bar{v}(\theta_i) = \mathbb{E}[v(X_i)]$, since $\mathbb{E}[v(X_i)]$ is a function of θ_i. Moreover, we adopt the DSIC constraint (2) and the ex post IR constraint (4). Then the welfare-maximizing problem is a linear program given by

$$\begin{aligned}
\underset{p,t}{\text{maximize}}\quad & \mathbb{E}[\textstyle\sum_i p_i(\theta)\bar{v}(\theta_i)] && \text{(7a)}\\
\text{subject to}\quad & p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) \\
& \geq p_i(\hat{\theta}_i, \theta_{-i})\bar{v}(\theta_i) - t_i(\hat{\theta}_i, \theta_{-i}),\ \forall i,\ \forall\theta,\ \forall\hat{\theta}_i, && \text{(7b)}\\
& p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) \geq 0,\ \forall i,\ \forall\theta, && \text{(7c)}\\
& \textstyle\sum_i p_i(\theta) \leq 1,\ \forall\theta, && \text{(7d)}\\
& p_i(\theta) \geq 0,\ \forall i,\ \forall\theta. && \text{(7e)}
\end{aligned}$$

It can be shown that a variant of the Vickrey-Clarke-Grove (VCG) mechanism [4], which we call the stochastic VCG mechanism, is an optimal solution to the welfare-maximizing problem (7).

Theorem 1: For each θ, let $\tilde{p}(\theta)$ be a solution to the following problem:

$$\begin{aligned}
\underset{p(\theta)}{\text{maximize}}\quad & \textstyle\sum_i p_i(\theta)\bar{v}(\theta_i) && \text{(8a)}\\
\text{subject to}\quad & \textstyle\sum_i p_i(\theta) \leq 1, && \text{(8b)}\\
& p_i(\theta) \geq 0,\ \forall i. && \text{(8c)}
\end{aligned}$$

For each i and each θ, let $\tilde{p}^{-i}(\theta)$ be a solution to the following problem:

$$\begin{aligned}
\underset{p(\theta)}{\text{maximize}}\quad & \textstyle\sum_j p_j(\theta)\bar{v}(\theta_i)\\
\text{subject to}\quad & \textstyle\sum_j p_j(\theta) \leq 1,\\
& p_i(\theta) = 0,\\
& p_j(\theta) \geq 0,\ \forall j \neq i.
\end{aligned}$$

For each i and each θ, define the payment scheme as

$$\tilde{t}_i(\theta) = \textstyle\sum_{j\neq i} \tilde{p}_j^{-i}(\theta)\bar{v}(\theta_j) - \sum_{j\neq i} \tilde{p}_j(\theta)\bar{v}(\theta_j). \quad (9)$$

Then the stochastic VCG mechanism $\tilde{\Gamma} = \{\tilde{p}, \tilde{t}\}$ is an optimal solution to the welfare-maximizing problem (7).

Proof: Since $\tilde{p}(\theta)$ maximizes (8a) subject to (8b) and (8c) for each θ, it follows that $\tilde{p}$ maximizes (7a) subject to (7d) and (7e). It remains to show that $\{\tilde{p}, \tilde{t}\}$ satisfies (7b) and (7c). Suppose (7b) is not satisfied for some i, θ and $\hat{\theta}_i$. Then

$$\tilde{p}_i(\theta)\bar{v}(\theta_i) - \tilde{t}_i(\theta) < \tilde{p}_i(\hat{\theta}_i, \theta_{-i})\bar{v}(\theta_i) - \tilde{t}_i(\hat{\theta}_i, \theta_{-i}).$$

Substituting from (9) for $\tilde{t}_i(\theta)$ and $\tilde{t}_i(\hat{\theta}_i, \theta_{-i})$, and noting that $\tilde{p}^{-i}(\theta) = \tilde{p}^{-i}(\hat{\theta}_i, \theta_{-i})$, we have

$$\textstyle\sum_j \tilde{p}_j(\theta)\bar{v}(\theta_j) < \sum_j \tilde{p}_j(\hat{\theta}_i, \theta_{-i})\bar{v}(\theta_j),$$

which contradicts $\tilde{p}(\theta)$ solving (8). To verify (7c), we have

$$\begin{aligned}
\tilde{p}_i(\theta)\bar{v}(\theta_i) - \tilde{t}_i(\theta) &= \textstyle\sum_j \tilde{p}_j(\theta)\bar{v}(\theta_j) - \sum_{j\neq i} \tilde{p}_j^{-i}(\theta)\bar{v}(\theta_j)\\
&= \textstyle\sum_j \tilde{p}_j(\theta)\bar{v}(\theta_j) - \sum_j \tilde{p}_j^{-i}(\theta)\bar{v}(\theta_j)\\
&\geq 0,
\end{aligned}$$

where the last inequality again follows from $\tilde{p}(\theta)$ solving (8). Therefore, the stochastic VCG mechanism $\tilde{\Gamma} = \{\tilde{p}, \tilde{t}\}$ solves the welfare-maximizing problem (7). ■

Note that the above implementation is very robust, in the following two aspects. First, each agent's best response depends on his own type only, whatever the others' types or bids; in particular, the best response does not depend on the prior distribution $\phi(\cdot)$. Second, the selection scheme $\tilde{p}$ does not depend on $\phi(\cdot)$ either, which means that this mechanism always works, whether the types are independent or not.

3) The Revenue-Maximizing Problem: We define the aggregator's revenue as the value derived from the generation of the selected provider minus the net payment made to the agents. Thus, we have the following the objective function:

$$\begin{aligned}
g(\theta, p, t) &= \mathbb{E}[\textstyle\sum_i (p_i(\theta)v(X_i) + t_i(\theta) - p_i(\theta)s_i(\theta, X_i))]\\
&= \textstyle\sum_i t_i(\theta).
\end{aligned}$$

Moreover, we adopt the DSIC constraint (2) and the interim IR constraint (5). Then the revenue-maximizing problem is a linear program given by:

$$\begin{aligned}
\underset{p,t}{\text{maximize}} \quad & \mathbb{E}[\textstyle\sum_i t_i(\theta)] & \text{(10a)}\\
\text{subject to} \quad & p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) \\
& \geq p_i(\hat{\theta}_i, \theta_{-i})\bar{v}(\theta_i) - t_i(\hat{\theta}_i, \theta_{-i}),\ \forall i,\ \forall \theta,\ \forall \hat{\theta}_i, & \text{(10b)}\\
& \mathbb{E}[p_i(\theta)\bar{v}(\theta_i) - t_i(\theta)|\theta_i] \geq 0,\ \forall i,\ \forall \theta_i, & \text{(10c)}\\
& \textstyle\sum_i p_i(\theta) \leq 1,\ \forall \theta, & \text{(10d)}\\
& p_i(\theta) \geq 0,\ \forall i,\ \forall \theta. & \text{(10e)}
\end{aligned}$$

Since (10c) is less constrained than (7c), it follows that the stochastic VCG mechanism is a feasible (though suboptimal in general) solution to the revenue-maximizing problem (10). To obtain the optimal solution, one can solve the linear program directly. It turns out that when there is "enough" correlation among the agents' types, the optimal mechanism has an appealing property, which we will discuss next.

III. Main Results

We first consider the optimal mechanism under full information, based on which we define full surplus extraction. We then define full correlation of the joint distribution of the types, under which the optimal mechanism for (10) extracts the full surplus. As theoretical contribution, we derive impossibility results to show that there do not exist "better" mechanisms.

A. Full Information and Full Correlation

When the aggregator has full information about θ, then the revenue-maximizing problem is a family of problems indexed by θ, where the IC constraints are discarded:

$$\begin{aligned}
\underset{p(\theta),t(\theta)}{\text{maximize}} \quad & \textstyle\sum_i t_i(\theta) & \text{(11a)}\\
\text{subject to} \quad & p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) \geq 0,\ \forall i, & \text{(11b)}\\
& \textstyle\sum_i p_i(\theta) \leq 1, & \text{(11c)}\\
& p_i(\theta) \geq 0,\ \forall i, & \text{(11d)}
\end{aligned}$$

which is equivalent to the following problem:

$$\begin{aligned}
\underset{p(\theta)}{\text{maximize}} \quad & \textstyle\sum_i p_i(\theta)\bar{v}(\theta_i) & \text{(12a)}\\
\text{subject to} \quad & \textstyle\sum_i p_i(\theta) \leq 1, & \text{(12b)}\\
& p_i(\theta) \geq 0,\ \forall i. & \text{(12c)}
\end{aligned}$$

On the other hand, this family of problems is also equivalent to a single optimization problem:

$$\begin{aligned}
\underset{p,t}{\text{maximize}} \quad & \mathbb{E}[\textstyle\sum_i t_i(\theta)] & \text{(13a)}\\
\text{subject to} \quad & p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) \geq 0,\ \forall i,\ \forall \theta, & \text{(13b)}\\
& \textstyle\sum_i p_i(\theta) \leq 1,\ \forall \theta, & \text{(13c)}\\
& p_i(\theta) \geq 0,\ \forall i,\ \forall \theta. & \text{(13d)}
\end{aligned}$$

In the remaining of the paper, we denote by $\Gamma^* = \{p^*, t^*\}$ an optimal solution to problem (13), which also solves problem (11) and problem (12).

Compared with problem (10), problem (13) omits the IC constraints, but strengthens the IR constraints. It is still true, though not obvious, that the optimal value of problem (10) is upper bounded by that of problem (13).

Proposition 1: Let $\check{\Gamma} = \{\check{p}, \check{t}\}$ be an optimal solution to problem (10). Then

$$\mathbb{E}[\textstyle\sum_i \check{t}_i(\theta)] \leq \mathbb{E}[\textstyle\sum_i t_i^*(\theta)].$$

Proof: We have

$$\begin{aligned}
\mathbb{E}[\textstyle\sum_i \check{t}_i(\theta)] &= \textstyle\sum_i \mathbb{E}[\mathbb{E}[\check{t}_i(\theta)|\theta_i]]\\
&\leq \textstyle\sum_i \mathbb{E}[\mathbb{E}[\check{p}_i(\theta)\bar{v}(\theta_i)|\theta_i]]\\
&= \textstyle\sum_i \mathbb{E}[\check{p}_i(\theta)\bar{v}(\theta_i)]\\
&= \mathbb{E}[\textstyle\sum_i \check{p}_i(\theta)\bar{v}(\theta_i)]\\
&\leq \mathbb{E}[\textstyle\sum_i p_i^*(\theta)\bar{v}(\theta_i)]\\
&= \mathbb{E}[\textstyle\sum_i t_i^*(\theta)],
\end{aligned}$$

where the first inequality follows from (10c); the second inequality follows from the fact that $p^*(\theta)$ is an optimal solution to problem (12) for each θ; and the last equality follows from the equivalence between problem (11) and problem (12). ■

Proposition 1 motivates the following definition.

Definition 1 (Full Surplus Extraction): A mechanism $\Gamma = \{p, t\}$ is said to extract the full surplus, if

$$\mathbb{E}[\textstyle\sum_i t_i(\theta)] = \mathbb{E}[\textstyle\sum_i t_i^*(\theta)].$$

Recall that we treat the joint distribution $\phi(\theta)$ and the conditional distribution $\phi_i(\theta_{-i}|\theta_i)$ as probability mass functions, and that $|\Theta_i| = m_i, |\Theta_{-i}| = n_i$. Define the conditional distribution matrix A_i for each i as

$$A_i = \begin{pmatrix} \phi_i(\theta_{-i}^1|\theta_i^1) & \cdots & \phi_i(\theta_{-i}^{n_i}|\theta_i^1)\\ \vdots & \ddots & \vdots \\ \phi_i(\theta_{-i}^1|\theta_i^{m_i}) & \cdots & \phi_i(\theta_{-i}^{n_i}|\theta_i^{m_i}) \end{pmatrix},$$

where the rows are indexed by the elements in Θ_i, and the columns are indexed by the elements in Θ_{-i}.

Definition 2 (Full Correlation): The joint distribution $\phi(\theta)$ is said to have full correlation, if A_i has rank m_i for all i.

One special case of full correlation is perfect correlation, i.e., for any i, knowing the value of θ_i gives the exact value of θ_{-i}. In that case, each A_i has exactly one entry 1 in each row and 0s elsewhere.

B. Optimal Mechanism with Types of Full Correlation

Under the full correlation condition, full surplus extraction is achievable. Interestingly, this can be proved by construction based on the stochastic VCG mechanism.

Theorem 2: If $\phi(\theta)$ has full correlation, the optimal mechanism for (10) extracts the full surplus.

Proof: Let $\tilde{\Gamma} = \{\tilde{p}, \tilde{t}\}$ be a stochastic VCG mechanism specified in Theorem 1. Define

$$h_i(\theta_i) = \textstyle\sum_{\theta_{-i}} \phi_i(\theta_{-i}|\theta_i)(\tilde{p}_i(\theta)\bar{v}(\theta_i) - \tilde{t}_i(\theta)).$$

Let $h_i = (h_i(\theta_i^1), \ldots, h_i(\theta_i^{m_i}))^\intercal$. Since A_i has rank m_i, there exists a $t_i' = (t_i'(\theta_{-i}^1), \ldots, t_i'(\theta_{-i}^{n_i}))^\intercal$ such that $A_i t_i' = h_i$.

Now let $p = \tilde{p}$ and $t_i(\theta) = \tilde{t}_i(\theta) + t'_i(\theta_{-i})$. We show that $\Gamma = \{p, t\}$ is an optimal solution to problem (10) that extracts the full surplus.

First, $\{p, t\}$ satisfies (10b), since $\{\tilde{p}, \tilde{t}\}$ satisfies (7b) and $t'_i(\theta_{-i})$ does not depend on θ_i. To verify (10c), we have

$$\begin{aligned} &\mathbb{E}[p_i(\theta)\bar{v}(\theta_i) - t_i(\theta)|\theta_i] \\ &= \textstyle\sum_{\theta_{-i}} \phi_i(\theta_{-i}|\theta_i)(\tilde{p}_i(\theta)\bar{v}(\theta_i) - \tilde{t}_i(\theta) - t'_i(\theta_{-i})) \\ &= \textstyle\sum_{\theta_{-i}} \phi_i(\theta_{-i}|\theta_i)(\tilde{p}_i(\theta)\bar{v}(\theta_i) - \tilde{t}_i(\theta)) \\ &\quad - \textstyle\sum_{\theta_{-i}} \phi_i(\theta_{-i}|\theta_i)t'_i(\theta_{-i}) \\ &= h_i(\theta_i) - h_i(\theta_i) \\ &= 0. \end{aligned}$$

Finally, the above equation and the equivalence between problem (8) and problem (12), shows that equalities hold everywhere in the proof of Proposition 1. Thus, $\{p, t\}$ extracts the full surplus. ■

Corollary 1: Let $I = 2$. Define the joint distribution matrix A as

$$A = \begin{pmatrix} \phi(\theta_1^1, \theta_2^1) & \cdots & \phi(\theta_1^1, \theta_2^{m_2}) \\ \vdots & \ddots & \vdots \\ \phi(\theta_1^{m_1}, \theta_2^1) & \cdots & \phi(\theta_1^{m_1}, \theta_2^{m_2}) \end{pmatrix}.$$

If A is square and invertible, the optimal mechanism for (10) extracts the full surplus.

Proof: Since A is invertible, $\det A \neq 0$. Note that A_1 is obtained by dividing each row m of A by the marginal distribution $\phi(\theta_1^m, \cdot)$. Thus, we have

$$\det A_1 = \frac{\det A}{\prod_{m=1}^{m_1} \phi(\theta_1^m, \cdot)} \neq 0.$$

So A_1 is invertible. Similarly, A_2 is also invertible, and the full correlation condition is satisfied. The claim follows. ■

C. Impossibility Results

Theorem 2 states that under certain conditions, there exists a DSIC and interim IR mechanism that extracts the full surplus. It is natural to ask whether there are mechanisms with more desirable properties. One direction is to strengthen the IR constraint, by replacing the interim IR constraint (10c) by the ex post IR constraint (4). It turns out that in general, this leads to an impossibility result, even if the IC constraint is relaxed.

Lemma 1: If a mechanism $\Gamma = \{p, t\}$ is ex post individual rational and extracts the full surplus, then $\{p, t\}$ must be a solution to problem (13).

Proof: Since $\Gamma = \{p, t\}$ is ex post IR, we have

$$p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) \geq 0,\ \forall i,\ \forall \theta,$$

or

$$\mathbb{E}[\textstyle\sum_i t_i(\theta)] \leq \mathbb{E}[\sum_i p_i(\theta)\bar{v}(\theta_i)].$$

Since $\Gamma = \{p, t\}$ extracts the full surplus, we have

$$\mathbb{E}[\textstyle\sum_i t_i(\theta)] = \mathbb{E}[\sum_i t_i^*(\theta)] = \mathbb{E}[\sum_i p_i^*(\theta)\bar{v}(\theta_i)].$$

Therefore,

$$\mathbb{E}[\textstyle\sum_i p_i^*(\theta)\bar{v}(\theta_i)] \leq \mathbb{E}[\sum_i p_i(\theta)\bar{v}(\theta_i)],$$

which means that $p(\theta)$ is a solution to problem (12) for each θ. It follows that $\{p, t\}$ is a solution to problem (13). ■

Assumption 1: There exists some i who has at least two types $\theta_i \neq \hat{\theta}_i$ such that $\bar{v}(\theta_i) > \bar{v}(\hat{\theta}_i)$ and $p_i^*(\hat{\theta}_i, \theta_{-i}) > 0$ for some θ_{-i}.

Theorem 3: Under Assumption 1, there does not exist a mechanism that is Bayesian incentive compatible, ex post individual rational, and extracts the full surplus.

Proof: Suppose such a mechanism exists. By Lemma 1, it must be a solution to problem (13), hence denoted by $\Gamma^* = \{p^*, t^*\}$. By Assumption 1, there exists some i with $\theta_i \neq \hat{\theta}_i$ such that $\bar{v}(\theta_i) > \bar{v}(\hat{\theta}_i)$ and $p_i^*(\hat{\theta}_i, \theta_{-i}) > 0$ for some θ_{-i}. Then

$$\begin{aligned} &\mathbb{E}[p_i^*(\hat{\theta}_i, \theta_{-i})\bar{v}(\theta_i) - t_i^*(\hat{\theta}_i, \theta_{-i})|\theta_i] \\ &> \mathbb{E}[p_i^*(\hat{\theta}_i, \theta_{-i})\bar{v}(\hat{\theta}_i) - t_i^*(\hat{\theta}_i, \theta_{-i})|\theta_i] \\ &= 0 \\ &= \mathbb{E}[p_i^*(\theta_i, \theta_{-i})\bar{v}(\theta_i) - t_i^*(\theta_i, \theta_{-i})|\theta_i], \end{aligned}$$

which violates the BIC constraint (3). Thus, there is no BIC and ex post IR mechanism that extracts the full surplus. ■

Alternatively, we can strengthen the definition of full surplus extraction. A mechanism $\Gamma = \{p, t\}$ is said to *ex post* extract the full surplus, if

$$\textstyle\sum_i t_i(\theta) = \sum_i t_i^*(\theta),\ \forall \theta.$$

Lemma 2: If a mechanism $\Gamma = \{p, t\}$ ex post extracts the full surplus, then it is ex post individual rational.

Proof: Since $\Gamma = \{p, t\}$ ex post extracts the full surplus, it solves (11) for each θ. Therefore,

$$p_i(\theta)\bar{v}(\theta_i) - t_i(\theta) = 0,\ \forall i,\ \forall \theta,$$

which implies the ex post IR. ■

Corollary 2: Under Assumption 1, there does not exist a mechanism that is Bayesian incentive compatible, and ex post extracts the full surplus.

Proof: It follows by Lemma 2 and Theorem 3. ■

Note that the impossibility results obtained so far do not depend on the structure of $\phi(\cdot)$, which assure us that Theorem 2 characterizes the best possible solution one could expect.

IV. Example

In this section, we validate Theorem 2 by providing a numerical example, which also illustrates the case where full surplus extraction is not achievable.

Among the various models of wind power generation [17], [18], [19], [20], we adopt a simple yet realistic model. Assume that agent i's generation is $X_i = \gamma_i W_i^3$, where γ_i captures his technology (such as the turbine size and the energy conversion efficiency), and W_i denotes the local wind speed. W_i is a Rayleigh distributed random variable with parameter $\sigma_i > 0$, whose probability density function is given by

$$f(x; \sigma_i) = (x/\sigma_i^2)e^{-x^2/(2\sigma_i^2)},\ x \geq 0.$$

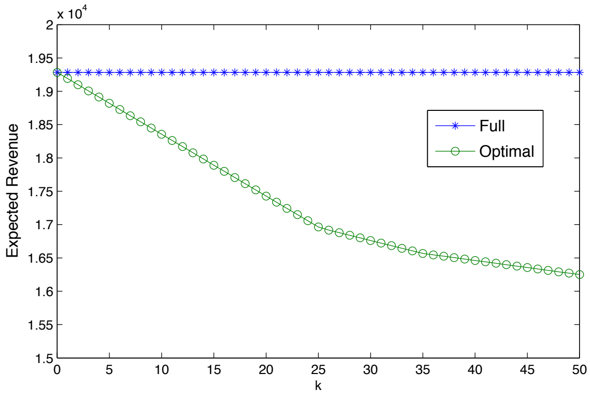

Fig. 2. The expected revenue of the full information mechanism for problem (13) and the optimal mechanism for problem (10) versus the uncertainty in γ_i's, whereas σ_i's are fully correlated.

We assume that σ_i's are fully correlated, whereas each γ_i is independent of γ_{-i} and σ. Thus, when γ is common knowledge, we have one-dimensional types $\theta_i = \sigma_i$ that are fully correlated; when each γ_i is private information, we have two-dimensional types $\theta_i = (\gamma_i, \sigma_i)$ that are not fully correlated. In the latter case, one cannot expect achieving the full surplus extraction, as verified by the following results.

Let $I = 2$ and $v(X_i) = X_i$, i.e., each unit of wind power is worth one unit of money. It can be shown that

$$\bar{v}(\theta_i) = \mathbb{E}_{X_i}[v(X_i)] = (3\sqrt{2\pi}/2)\gamma_i \sigma_i^3,$$

or simply $\bar{v}(\theta_i) = \gamma_i \sigma_i^3$ by normalization. Assume that $\sigma_1 \in \{\sigma_1^L = 1, \sigma_1^M = 3, \sigma_1^H = 5\}, \sigma_2 \in \{\sigma_2^L = 2, \sigma_2^M = 4, \sigma_2^H = 6\}$, and the joint distribution matrix for σ is

$$\begin{array}{c} \\ \sigma_1^L \\ \sigma_1^M \\ \sigma_1^H \end{array} \begin{array}{c} \begin{array}{ccc} \sigma_2^L & \sigma_2^M & \sigma_2^H \end{array} \\ \begin{pmatrix} .25 & .05 & 0 \\ .05 & .30 & .05 \\ 0 & .05 & .25 \end{pmatrix} \end{array}.$$

That is, the levels of wind speed at those two locations are highly positively correlated. It is easy to check that this matrix is invertible, so that σ_1 and σ_2 are fully correlated by Corollary 1. In the k-th scenario, γ_i is independently distributed as

$$\gamma_i = \begin{cases} \bar{\gamma}_i + k & \text{with probability } 1/2 \\ \bar{\gamma}_i - k & \text{with probability } 1/2 \end{cases},$$

where $\bar{\gamma}_1 = 190, \bar{\gamma}_2 = 200$. Here k characterizes the deviation of γ_i. When $k = 0$, the types $\theta_i = \sigma_i$ are fully correlated; when $k > 0$, the types $\theta_i = (\gamma_i, \sigma_i)$ are not fully correlated.

We solve the linear programs (10) and (13) and compare the expected revenue, as shown in Fig. 2. Since the expected value of γ_i does not change, the expected revenue under full information remains the same as k varies. When $k = 0$, the optimal mechanism for problem (10) extracts the full surplus; as k increases, its expected revenue decreases, since the uncertainty of γ_i to the aggregator increases.

V. Conclusion

We have formulated a unified optimization problem for stochastic resource auction design. The stochastic VCG mechanism is a robust implementation for the welfare-maximizing objective. For the revenue-maximizing objective, under the full correlation condition, there exists a dominant strategy incentive compatible and interim individual rational mechanism that extracts the full surplus. The impossibility results show that mechanisms with stronger properties do not exist.

As the numerical example suggests, it is an interesting topic to quantify the revenue loss against the full information case, when the types are not fully correlated. One can also consider multiple winners and continuous types for extensions. In future work, we will investigate a dynamic version of this problem, by employing the dynamic mechanism design theory.

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