Market Mechanisms for Buying Random Wind

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***Abstract*—The intermittent nature of wind power leads to the question of how wind power producers can participate in a deregulated electricity market. In the proposed auction paradigm, wind farms bid probability distributions of generation, instead of bidding cost functions as thermal units do. Our focus is to design incentive compatible mechanisms that elicit truthful information of strategic agents who supply stochastic resource. We first study the aggregator's problem of how to select the wind farms, which have the most desirable distributions. We then study the independent system operator's (ISO's) problem of how to price wind energy for stochastic economic dispatch.**

***Index Terms*—Game theory, mechanism design, renewable energy integration, smart grid, stochastic resource auction.**

I. Introduction

WITH THE increasing penetration of wind power, challenges arise in integrating such random generation into the current electricity grid and market. While wind power producers are commonly treated as negative loads and receive feed-in tariffs, this scheme would be no longer applicable when the penetration level is high with substantial reserve margin needed.

Alternatively, wind power producers can be required to participate in the competitive electricity pool through a two-settlement system. In the *ex-ante* day-ahead (DA) forward market, the wind power producer commits to a fixed amount of generation; in the ex-post real-time (RT) spot market, it pays a penalty for the shortfall if any. However, it still remains unclear what are the appropriate market mechanisms.

In the literature, the determination of the committed quantity can be posed as an optimization problem [2, Ch. 7], [3]. In [4], they introduce risky power contracts in addition to firm power contracts to enable flexible and efficient wind power aggregation. They focus on how optimal offerings and equilibria depend on exogenous price signals, and on deriving concept and expressions for critical prices from the perspective of wind power producers. To address the risk of not meeting operating constraints such as power balance due to the uncertainty of wind power generation, a new paradigm for power system operation called risk-limiting dispatch has been proposed [5]. That paradigm employs RT information about supply and demand, taking into account the stochastic nature of wind power generation, and determines a risk-constrained stochastic optimal dispatch.

There are two major issues of the existing approaches. 1) It is assumed that wind power producers have no market power to influence the market prices when they determine the optimal trading strategies. That assumption does not hold with a high penetration of wind power. 2) The optimal dispatch under uncertainty requires probabilistic models of wind power generation, which may not be revealed truthfully with strategic wind power producers.

In this work, we observe that it may not be efficient for wind power producers to bid cost functions as thermal units do. Instead, bidding probability distributions of generation can potentially exploit the value of aggregation. This motivates the proposed stochastic resource auction paradigm. The underlying assumption is that the current two-settlement system does not change, but aggregators are allowed to enter the market, who procure wind power through auctions and assume any risk due to the uncertainty of wind power generation. The key feature of a stochastic resource auction is that while a wind farm's realized generation is random, the probability distribution can be learned beforehand as its private information.

Auction and market design for electricity markets is a well-studied problem [6]–[10]. However, almost all of economic and auction theory deals with classical goods. The auction design problem, we introduce for stochastic goods, has received only scant attention, if at all. Our focus is to design incentive compatible mechanisms that elicit truthful information of strategic agents who supply stochastic resource. We will explore the problem from both an aggregator's and an independent system operator's (ISO's) points of view.

We note that following the same motivation, one can approach the same problem in the contract design framework that typically considers a principal and a single agent [11], [12]. In [13], they consider an aggregator procuring both conventional and renewable powers from a single producer that has multidimensional private information, and show that the optimal mechanism is a menu of contracts. As another context, [14] proposes a dynamic contract design problem with an application to indirect load control. The key characteristic of their setting is that the principal has no capability to monitor the agent's control or the state of the engineered system, whereas the agent has perfect observations.

This paper is organized as follows. In Section II, we make an argument why it is more efficient for wind power producers to bid probability distribution of generation rather than

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cost functions, and introduce the stochastic resource auction paradigm. In Section III, we study the aggregator's problem how to select the wind farms, which have the most desirable distributions. In Section IV, we study the ISO's problem how to price wind energy for stochastic economic dispatch. We then present a case study to complement the previous analysis in Section V. Section VI concludes this paper.

II. Problem Statement

A. Model

We consider a single-period model composed of a DA market and an RT market. There are N wind farms as strategic players (or agents), indexed by $i = 1, \ldots, N$. The wind power generation of agent i at the RT market is a random variable $W_i \in [0, \bar{w}]$, where $\bar{w}$ is a constant denoting the maximum capacity over all agents. Denote by w_i the realization of W_i. Throughout this paper, we assume that W_i's are independent. The probability distribution of W_i is parametrized by σ_i, referred to as agent i's type, which is a one- or multidimensional parameter learned privately by agent i at the DA market.

Example 1: Let the wind power generation W_i's be normalized by the maximum nominal capacity over all agents, so that $\bar{w} = 1$. When W_i has a beta distribution, σ_i is a two-dimensional (2-D) parameter (α_i, β_i), where $\alpha_i > 0, \beta_i > 0$.

B. Bidding Cost Functions

At the DA market, wind farms can bid cost functions as thermal units do. While the marginal cost is negligible, the generation is random, so that agent i incurs a balancing cost as a function of the offered quantity x_i. Let $p > 0$ be the cost of load shedding. Then, the cost function of agent i is

$$C_i(x_i) = \mathbb{E}[p(x_i - W_i)_+]$$

where we define $x_+ = \max\{x, 0\}$. Note that any excess generation $(W_i - x_i)_+$ is assumed to be spilled.

The cost function $C_i(\cdot)$ is well behaved. First, it is increasing. Moreover, it is convex, as the expectation of a convex function. In this market architecture, the agents bear the risk, since it is their own responsibility to fulfill the committed quantity.

C. Bidding Probability Distributions

Alternatively, there can be an aggregator who aggregates wind power generation and bears the risk of buying power at the RT market to make up for any shortfall. In this market architecture, the wind farms can bid probability distributions (or equivalently, σ_i's), so that they do not need to make an estimate of p. The cost function of the aggregator is

$$C(x) = \mathbb{E}[p(x - \sum_i W_i)_+].$$

The following result implies that bidding probability distributions are more efficient than bidding cost functions.

Proposition 1: For any given $x \geq 0$, let $(x_1^*, \ldots, x_N^*)$ be an optimal solution to the following problem:

$$\begin{aligned} \underset{x_1,\ldots,x_N}{\text{minimize}} \quad & \textstyle\sum_i C_i(x_i) \\ \text{subject to} \quad & \textstyle\sum_i x_i = x \\ & x_i \geq 0 \quad \forall i. \end{aligned}$$

Then,

$$C(x) \leq \sum_i C_i(x_i^*).$$

Proof: We have

$$\begin{aligned} C(x) &= \mathbb{E}[p(\textstyle\sum_i x_i^* - \sum_i W_i)_+] \\ &\leq \textstyle\sum_i \mathbb{E}[p(x_i^* - W_i)_+] \\ &= \textstyle\sum_i C_i(x_i^*) \end{aligned}$$

where the inequality follows from the fact that $(x+y)_+ \leq x_+ + y_+$ for any x and y. ■

Therefore, bidding probability distributions can potentially exploit the value of aggregation and reduce the social cost. The surplus $\sum_i C_i(x_i^*) - C(x)$ provides incentives for aggregators to enter the market who bear the risk due to the uncertainty of wind power generation.

Example 2: Let $p = 1$, $\bar{w} = 1$, $x = 1$, and $f_1(w) = f_2(w) = 1, w \in [0, 1]$, where $f_i(\cdot)$ is the probability density function of W_i. When the wind farms bid cost functions, we have $x_1^* = x_2^* = 1/2$, so that $C_1(x_1^*) = C_2(x_2^*) = 1/8$. When they bid probability distributions, since the probability density function of $W_1 + W_2$ is

$$f(w) = \begin{cases} w, & w \in [0, 1] \\ 2 - w, & w \in (1, 2] \end{cases}$$

we have $C(x) = 1/6$. Thus, bidding probability distributions reduce cost by

$$1/8 + 1/8 - 1/6 = 1/12.$$

D. Stochastic Resource Auction

Proposition 1 motivates a new paradigm for wind power procurement, referred to as the stochastic resource auction, in which wind farms bid probability distributions.

The timeline of the stochastic resource auction is shown in Fig. 1. At the DA market, each agent i learns its own type σ_i and submits a bid $\hat{\sigma}_i$, which could be different from the true type σ_i. Based on the bid profile $\hat{\sigma} = (\hat{\sigma}_1, \ldots, \hat{\sigma}_N)$, M out of the N agents are selected as the wind power providers, the set of which is denoted by $\mathcal{I}$. The unselected agents get zero payoffs and leave the auction. Each selected agent i makes a payment $t_{d,i}(\hat{\sigma})$ to the aggregator. At the RT market, upon the realization of W_i, each selected agent i gets paid $t_{r,i}(\hat{\sigma}, w_i)$ from the aggregator. Note that we allow $t_{d,i}$ or $t_{r,i}$ to be negative, indicating a payment in the reverse direction. Therefore, the selection and payment schemes define a direct revelation mechanism $\Gamma = \{\mathcal{I}, t_{d,i}, t_{r,i}\}$. The payoff of agent $i \in \mathcal{I}$ is given by

$$\pi_i(\hat{\sigma}, w_i) = t_{r,i}(\hat{\sigma}, w_i) - t_{d,i}(\hat{\sigma}).$$

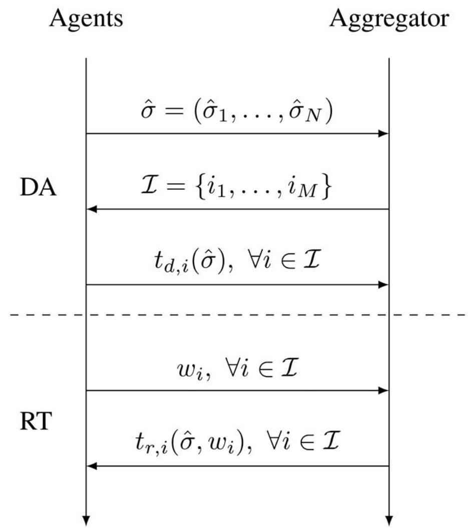

Fig. 1. Timeline of the stochastic resource auction.

We will consider two settings in this paper. In the first setting, the wind farms do not directly participate in the electricity pool; instead, an aggregator selects some of them as the wind power providers who have the most desirable distributions. We refer to this as the aggregator's problem, in which $M < N$. In the second setting, all the wind farms are online, and the problem is how to price wind energy for stochastic economic dispatch. We refer to this as the ISO's problem, in which $M = N$.

In both settings, the aggregator or the ISO needs to know the probability distributions of wind power generation to find the best set of wind power providers or to make an optimal dispatch. Since the agents are strategic, they may not have incentives to reveal their private information truthfully. Our goal is to design incentive compatible mechanisms to elicit the true types of the agents. The focus of this paper is welfare maximization, rather than profit maximization which is left for future work.

We summarize the notations in Table I. For simplicity, we will assume that there is exactly one wind farm located at each bus in the ISO's problem, so that both agents and buses are indexed by i. The reader can consult [15] for the standard game-theoretic terminology used throughout this paper.

III. Aggregator's Problem: Selecting Wind Power Providers

We first consider a basic scenario where a single agent is selected as the provider, for technical or regulatory reasons. We will make several generalizations afterward.

In the simplest scenario, the objective is to identify the agent who yields the highest expected generation. That is, the aggregator problem is

$$\underset{i}{\text{maximize}} \ \mathbb{E}[W_i].$$

Since the resource is stochastic, we derive a variant of the well-known Vickrey–Clarke–Groves (VCG) mechanism [15]. We then propose an alternative mechanism which has a commitment-with-penalty payment structure [3].

TABLE I
NOMENCLATURE

N	Number of agents
M	Number of selected agents
$\mathcal{I}$	Set of selected agents
W_i	Random generation of agent i
$\hat{W}_i$	Reported random generation of agent i
w_i	Realization of W_i
σ_i	Parametrized probability distribution of W_i
$\hat{\sigma}_i$	Parametrized probability distribution of $\hat{W}_i$
f_i	Probability density function of W_i
$t_{d,i}$	DA payment made by agent i
$t_{r,i}$	RT payment made to agent i
π_i	Payoff of agent i
g	Objective function
p_i	Cost of load shedding at bus i
λ	Contract price in the proposed mechanisms
$\lambda_{d,i}$	LMP at the DA market at bus i
$\lambda_{r,i}$	LMP at the RT market at bus i
μ	Penalty price in the proposed mechanisms
C_i	Cost function of agent i
c_i	Competitive cost function of thermal units at bus i
B_{ij}	(i,j)–th element of the bus susceptance matrix
$\bar{f}_{ij}$	Flow limit of branch i–j
D_i	Demand at bus i
x_i	Thermal power production at bus i
y_i	Wind power production scheduled at bus i
$\theta_{d,i}$	Voltage phase angle at the DA market at bus i
y_i^+	Power procured at the RT market at bus i
y_i^-	Power spilled at the RT market at bus i
$\theta_{r,i}$	Voltage phase angle at the RT market at bus i

A. Stochastic VCG Mechanism

Denote by $\hat{W}_i$ the random variable parametrized by $\hat{\sigma}_i$. The selection scheme $\mathcal{I} = \{i\}$ is given by

$$i \in \arg\max_j \mathbb{E}[\hat{W}_j] \tag{1}$$

where any tie-breaking rule applies. That is, the one who claims to yield the highest expected generation is selected. This makes sense providing that the incentive compatibility is achieved. Moreover, it is natural to specify

$$t_{r,i}(\hat{\sigma}, w_i) = \lambda w_i \tag{2}$$

where $\lambda > 0$ is an adjustable parameter, denoting the price the aggregator pays the selected agent for the realized generation, which does not need to be the price at the DA market, nor that at the RT market. It remains to specify $t_{d,i}(\hat{\sigma})$ to achieve the incentive compatibility.

We have

$$\mathbb{E}[\pi_i(\hat{\sigma}_i, \hat{\sigma}_{-i}, W_i)] = \lambda \mathbb{E}[W_i] - t_{d,i}(\hat{\sigma}_i, \hat{\sigma}_{-i}).$$

Fix $\hat{\sigma}_{-i}$. On one hand, if $\mathbb{E}[W_i] \geq \mathbb{E}[\hat{W}_j]$ for all $j \neq i$, then

$$t_{d,i}(\sigma_i, \hat{\sigma}_{-i}) \leq t_{d,i}(\hat{\sigma}_i, \hat{\sigma}_{-i})$$

for any $\hat{\sigma}_i$ such that $\mathbb{E}[\hat{W}_i] \geq \mathbb{E}[\hat{W}_j]$ for all $j \neq i$. On the other hand, if $\mathbb{E}[W_i] < \mathbb{E}[\hat{W}_j]$ for some j, then

$$\lambda \mathbb{E}[W_i] - t_{d,i}(\hat{\sigma}_i, \hat{\sigma}_{-i}) \leq 0$$

for any $\hat{\sigma}_i$ such that $\mathbb{E}[\hat{W}_i] \geq \mathbb{E}[\hat{W}_j]$ for all $j \neq i$. Thus, a candidate DA payment scheme can be

$$t_{d,i}(\hat{\sigma}) = \lambda \max_{j \neq i} \mathbb{E}[\hat{W}_j] \tag{3}$$

which also ensures the individual rationality.

The proposed mechanism is referred to as the stochastic VCG (SVCG) mechanism, since it is an analog of the standard VCG mechanism. The expected RT payment $\lambda \mathbb{E}[W_i]$ can be viewed as the counterpart of the valuation in the standard VCG mechanism, and the DA payment $\lambda \max_{j \neq i} \mathbb{E}[\hat{W}_j]$ as the counterpart of the usual payment (or externality). We now formally state the properties of this mechanism.

Theorem 1: The SVCG mechanism specified by (1)–(3) is efficient, dominant strategy incentive compatible, and individual rational.

Proof: The selection scheme (1) implies that the SVCG mechanism is efficient.

Fix an agent i and $\hat{\sigma}_{-i}$. We show that bidding σ_i is the best response of agent i. Note that conditioned on that agent i is selected, its expected payoff is independent of $\hat{\sigma}_i$

$$\mathbb{E}[\pi_i(\hat{\sigma}_i, \hat{\sigma}_{-i}, W_i)] = \lambda \mathbb{E}[W_i] - \lambda \max_{j \neq i} \mathbb{E}[\hat{W}_j].$$

Suppose that agent i is selected by bidding σ_i

$$\mathbb{E}[W_i] \geq \max_{j \neq i} \mathbb{E}[\hat{W}_j]$$

so that its current expected payoff is nonnegative. Consider that it changes the bid to $\hat{\sigma}_i$. If it is still selected, its expected payoff remains the same. If it is no longer selected, it gets a zero payoff. In either case, its expected payoff does not increase.

Suppose that agent i is not selected by bidding σ_i

$$\mathbb{E}[W_i] \leq \max_{j \neq i} \mathbb{E}[\hat{W}_j]$$

so that its current payoff is zero. Consider that it changes the bid to $\hat{\sigma}_i$. If it is not selected, it still gets a zero payoff. If it is selected, its expected payoff is nonpositive. In either case, its expected payoff does not increase.

Thus, truth-telling is a dominant strategy for each agent, which means that the SVCG mechanism is dominant strategy incentive compatible.

Since the expected payoff for each agent is nonnegative, the SVCG mechanism is individual rational. ■

Remark 1: The SVCG mechanism can be interpreted in the following way. At the DA market, the provider makes a contractual payment $\lambda \max_{j \neq i} \mathbb{E}[\hat{W}_j]$ to the aggregator, which depends on the reported expected generation of the second highest bidder. At the RT market, the aggregator makes a payment $\lambda \mathbb{E}[W_i]$ to the provider for the actual generation at a price l. Dominant strategy incentive compatibility means that each agent will truthfully reveal its probability distribution, regardless of what the other agents do. Consequently, truth-telling for all agents is a dominant strategy equilibrium.

B. Stochastic Shortfall Penalty Mechanism

Since the wind farms have no intrinsic valuation for the wind power generation they produce, there exists a richer class of incentive compatible mechanisms beyond the standard mechanism design framework. We now propose an alternative mechanism with natural interpretations.

Recall that $W_i \in [0, \bar{w}]$ for all i. Ideally, if $W_i = \bar{w}$ with probability 1, then agent i is the desired provider. This suggests that $\bar{w}$ can serve as a reference point in a commitment-with-penalty payment structure. In the proposed mechanism, referred to as the stochastic shortfall penalty (SSP) mechanism, the selection scheme is the same as before

$$i \in \arg\max_j \mathbb{E}[\hat{W}_j]. \tag{4}$$

The DA payment scheme is given by

$$t_{d,i}(\hat{\sigma}) = -\lambda \bar{w}. \tag{5}$$

The RT payment scheme is given by

$$t_{r,i}(\hat{\sigma}, w_i) = -\mu(\hat{\sigma})(\bar{w} - w_i) \tag{6}$$

where $\mu(\cdot)$ is a function of $\hat{\sigma}$. The interpretation is the following. At the DA market, the aggregator makes a payment to the provider for the full capacity generation $\bar{w}$ at a price l. At the RT market, the provider is penalized for the shortfall $\bar{w} - w_i$ at a penalty price $\mu(\hat{\sigma})$. We specify

$$\mu(\hat{\sigma}) = \frac{\lambda \bar{w}}{\bar{w} - \max_{j \neq i} \mathbb{E}[\hat{W}_j]}. \tag{7}$$

Note that $\mu(\hat{\sigma}) \geq \lambda$, which is necessary for the mechanism to be incentive compatible.

Theorem 2: The SSP mechanism specified by (4)–(7) is efficient, dominant strategy incentive compatible, and individual rational.

Proof: The selection scheme (4) implies that the SSP mechanism is efficient.

Fix an agent i and $\hat{\sigma}_{-i}$. We show that bidding σ_i is the best response of agent i. Note that conditioned on that agent i is selected, its expected payoff is independent of $\hat{\sigma}_i$

$$\begin{aligned}\mathbb{E}[\pi_i(\hat{\sigma}_i, \hat{\sigma}_{-i}, W_i)] &= \lambda \bar{w} - \mu(\hat{\sigma})(\bar{w} - \mathbb{E}[W_i]) \\ &= \frac{\lambda \bar{w}(\mathbb{E}[W_i] - \max_{j \neq i} \mathbb{E}[\hat{W}_j])}{\bar{w} - \max_{j \neq i} \mathbb{E}[\hat{W}_j]}.\end{aligned}$$

Suppose that agent i is selected by bidding σ_i

$$\mathbb{E}[W_i] \geq \max_{j \neq i} \mathbb{E}[\hat{W}_j]$$

so that its current expected payoff is nonnegative. Consider that it changes the bid to $\hat{\sigma}_i$. If it is still selected, its expected payoff remains the same. If it is no longer selected, it gets a zero payoff. In either case, its expected payoff does not increase.

Suppose that agent i is not selected by bidding σ_i

$$\mathbb{E}[W_i] \leq \max_{j \neq i} \mathbb{E}[\hat{W}_j]$$

so that its current payoff is zero. Consider that it changes the bid to $\hat{\sigma}_i$. If it is not selected, it still gets a zero payoff. If it is selected, its expected payoff is nonpositive. In either case, its expected payoff does not increase.

Thus, truth-telling is a dominant strategy for each agent, which means that the SSP mechanism is dominant strategy incentive compatible.

Since the expected payoff for each agent is nonnegative, the SSP mechanism is individual rational. ■

Remark 2: There is a duality between those two mechanisms: At the DA market, money flows from the provider to the aggregator in SVCG (which depends on the second highest bid), whereas it flows from the aggregator to the provider in SSP (which is a constant). At the RT market, money flows from the aggregator to the provider in SVCG (which depends on the realization), whereas it flows from the provider to the aggregator in SSP (which depends on both the second highest bid and the realization). In the remaining of this paper, we will focus on the SVCG mechanism.

C. Generalizations

1) General Objective Function: In the basic scenario, the objective is to identify the agent who yields the highest expected generation. Now, we generalize the objective. The aggregator's problem is

$$\underset{i}{\text{maximize}}\ \mathbb{E}[g(W_i)]$$

where $g : [0, \bar{w}] \to \mathbb{R}$ is referred to as the objective function. We assume that $g(\cdot)$ is continuous, which satisfies most practical purposes, so that it attains a maximum and a minimum. We can consider $-g(\cdot)$ when we have a minimization problem. We give some examples of the objective function.

Example 3: When $g(w) = w$, we recover the basic scenario.

Example 4: When $g(w) = \min\{w, D\}$, the interpretation is that there is an upper bound $D \in [0, \bar{w}]$ on the demand at the RT market, beyond which the aggregator does not care about how much more power would be generated.

We now generalize the SVCG mechanism. The selection scheme is given by

$$i \in \arg\max_{j} \mathbb{E}[g(\hat{W}_j)]. \tag{8}$$

The payment schemes are given by

$$t_{d,i}(\hat{\sigma}) = \max_{j \neq i} \mathbb{E}[g(\hat{W}_j)],\ t_{r,i}(\hat{\sigma}, w_i) = g(w_i). \tag{9}$$

We state the properties of this mechanism in the following theorem. The proof is similar as before.

Theorem 3: The generalized SVCG mechanism specified by (8)–(9) is efficient, dominant strategy incentive compatible, and individual rational.

2) Selecting Multiple Providers: We now consider the scenario in which M $(< N)$ agents are selected according to the criterion represented by a continuous objective function $g : [0, \bar{w}]^M \to \mathbb{R}$. That is, the aggregator's problem is

$$\underset{i_1,\dots,i_M}{\text{maximize}}\ \mathbb{E}[g(W_{i_1}, \dots, W_{i_M})].$$

We give some examples of the objective function.

Example 5: When $g(w_{i_1}, w_{i_2}) = (w_{i_1} - w_{i_2})^2$, the interpretation is that the aggregator wants to contract with two agents who have the closest distributions in a certain sense.

Example 6: When $g(w_{i_1}, \dots, w_{i_M}) = w_{i_1} + \cdots + w_{i_M}$, the interpretation is that the aggregator wants to contract with M agents who yield the highest expected generation.

Theorem 4: Let the selection scheme be

$$(i_1, \dots, i_M) \in \arg\max_{j_1,\dots,j_M} \mathbb{E}[g(\hat{W}_{j_1}, \dots, \hat{W}_{j_M})].$$

For all $k = 1, \dots, M$, let the DA payment be

$$t_{d,i_k}(\hat{\sigma}) = \max_{j_1 \neq i_k,\dots,j_M \neq i_k} \mathbb{E}[g(\hat{W}_{j_1}, \dots, \hat{W}_{j_M})]$$

and the RT payment be

$$t_{r,i_k}(\hat{\sigma}, w_{i_k}) = \mathbb{E}[g(\hat{W}_{i_1}, \dots, \hat{W}_{i_M}) | \hat{W}_{i_k} = w_{i_k}].$$

Then, the proposed mechanism is efficient, dominant strategy incentive compatible, and individual rational.

Remark 3: For certain objective functions, the proposed mechanism may have an explicit form. Consider Example 6. Rank the agents in order of the reported expected generation

$$\mathbb{E}[\hat{W}_{i_1}] \geq \cdots \geq \mathbb{E}[\hat{W}_{i_N}].$$

Then agents $i_1, \dots, i_M$ are selected. For all $k = 1, \dots, M$, the payment schemes are given by

$$t_{d,i_k}(\hat{\sigma}) = \mathbb{E}[\hat{W}_{i_{M+1}}],\ t_{r,i_k}(\hat{\sigma}, w_{i_k}) = w_{i_k}.$$

3) Inclusion of Fixed Cost: While the marginal cost is negligible, each agent i may have a fixed cost $\gamma_i \geq 0$ when it is selected and produces generation. In this case, agent i's type is a pair (σ_i, γ_i), and the reported type is denoted by $(\hat{\sigma}_i, \hat{\gamma}_i)$. Let $\hat{\gamma} = (\hat{\gamma}_1, \dots, \hat{\gamma}_N)$. The payment schemes may also depend on $\hat{\gamma}$, and the payoff of a selected agent i is given by

$$\pi_i(\hat{\sigma}, \hat{\gamma}, w_i) = t_{r,i}(\hat{\sigma}, \hat{\gamma}, w_i) - t_{d,i}(\hat{\sigma}, \hat{\gamma}) - \gamma_i.$$

We can employ the same idea as before to design incentive compatible mechanisms. To illustrate, consider the following aggregator's problem:

$$\underset{i}{\text{maximize}}\ \lambda\mathbb{E}[W_i] - \gamma_i.$$

Theorem 5: Let the selection scheme be

$$i \in \arg\max_{j} \mathbb{E}[\hat{W}_j] - \hat{\gamma}_j.$$

Let the payment schemes be

$$t_{d,i}(\hat{\sigma}, \hat{\gamma}) = \max_{j \neq i} \lambda\mathbb{E}[\hat{W}_j] - \hat{\gamma}_j,\ t_{r,i}(\hat{\sigma}, \hat{\gamma}, w_i) = \lambda w_i.$$

Then, the proposed mechanism is efficient, dominant strategy incentive compatible, and individual rational.

Remark 4: The fixed cost γ_i is typically the operating cost. But, it may also be interpreted as the reservation utility, e.g., the expected payoff when agent i participates in the market in the conventional way. In that case, the proposed mechanism ensures that each agent voluntarily participates in the auction before seeking outside options.

IV. ISO's Problem: Pricing Wind Energy

Economic dispatch is the short-term determination of the optimal generation schedule to meet the system load, subject to the transmission constraints. In a two-settlement market system, such market clearing can be based on a two-stage stochastic programming approach [2], taking into account the uncertainty of wind power generation. We refer to this problem as the stochastic economic dispatch problem.

A. *Stochastic Economic Dispatch*

We consider a connected power network, which consists of N buses indexed by $i = 1, \ldots, N$, with agent i located at bus i. We assume that the supply from the thermal units at each bus i is competitive, represented by a cost function $c_i(\cdot)$, which is increasing, convex, and differentiable. The demand at each bus i is inelastic, denoted by $D_i \geq 0$.

For analytical and computational simplicity, we adopt a dc power flow model as a common practice [16]. In the dc flow model, a branch $i - j$ is characterized by B_{ij}, the negative of its susceptance, which satisfies $B_{ij} = B_{ji} \geq 0$. Let θ_i be the voltage phase angle at bus i. Let $\bar{f}_{ij}$ be the flow limit of branch $i - j$, which satisfies $\bar{f}_{ij} = \bar{f}_{ji} \geq 0$. Then, the active power flow over branch $i - j$ is given by

$$f_{ij} = B_{ij}(\theta_i - \theta_j) \leq \bar{f}_{ij}.$$

Let x_i and y_i be the thermal and wind power production scheduled at the DA market at bus i, and $\theta_{d,i}$ be the voltage phase angle. Let $x = (x_1, \ldots, x_N)$ and similarly for other variables. At the DA market, the ISO determines an optimal dispatch (x, y, θ_d) that minimizes the expected social cost

$$\underset{x,y,\theta_d}{\text{minimize}} \quad \sum\nolimits_i c_i(x_i) + \mathbb{E}[Q(y, \theta_d, W)] \tag{10a}$$

$$\text{subject to} \quad x_i + y_i - D_i = \sum\nolimits_j B_{ij}(\theta_{d,i} - \theta_{d,j}) \quad \forall i \tag{10b}$$

$$B_{ij}(\theta_{d,i} - \theta_{d,j}) \leq \bar{f}_{ij} \quad \forall (i,j) \tag{10c}$$

$$x_i, y_i \geq 0 \quad \forall i \tag{10d}$$

where $Q(y, \theta_d, w)$ is the recourse cost due to the uncertainty of wind power generation.

The recourse cost $Q(y, \theta_d, w)$ is the minimum cost to balance the power throughout the network at the RT market. Similarly as before, let $p_i > 0$ be the cost of load shedding at bus i and assume that any excess power is spilled. Let y_i^+ and y_i^- be the amount of power procured and disposed, respectively, at bus i, and $\theta_{r,i}$ be the voltage phase angle. Let $y^+ = (y_1^+, \ldots, y_N^+)$ and similarly for other variables. At the RT market, the ISO determines an optimal dispatch (y^+, y^-, θ_r) that minimizes the balancing cost

$$\underset{y^+,y^-,\theta_r}{\text{minimize}} \quad \sum\nolimits_i p_i y_i^+ \tag{11a}$$

$$\text{subject to} \quad w_i + y_i^+ - y_i^- - y_i = \sum\nolimits_j B_{ij}(\theta_{r,i} - \theta_{r,j} - \theta_{d,i} + \theta_{d,j}) \quad \forall i \tag{11b}$$

$$B_{ij}(\theta_{r,i} - \theta_{r,j}) \leq \bar{f}_{ij} \quad \forall (i,j) \tag{11c}$$

$$y_i^+, y_i^- \geq 0 \quad \forall i. \tag{11d}$$

The optimal value of (11) gives the recourse cost $Q(y, \theta_d, w)$.

Thus, we have formulated a two-stage stochastic programming problem (10)–(11). Moreover, it can be shown that the the first-stage problem (10) is a convex optimization problem.

B. *Pricing Mechanisms for Wind Power Producers*

To solve the problem (10)–(11) exactly, the ISO needs to know the true probability distributions of wind power generation. This can be potentially addressed through an incentivized pricing mechanism.

Traditionally, locational marginal pricing (LMP) is widely used as a settlement scheme for economic dispatch. In the two-settlement market with stochastic production, an extended LMP mechanism is proposed in [2, Ch. 3]. Specifically, the LMPs at the DA (or RT) market are the dual variables associated with the bus power balance equation (10b) [or (11b)]. Those mechanisms are efficient in a competitive environment when agents are price takers. For strategic agents, however, such LMP-based mechanisms may not incentivize them to reveal their characteristics truthfully, as illustrated in the following example.

Example 7: There is a single bus. The cost function of thermal power is $c(x) = x^2$. The probability distribution of W is given by the probability density function $f(w) = 1, w \in [0, 1]$. The demand is $D = 1$. The cost of load shedding is $p = 2$. When the true distribution is revealed, the ISO solves the following problem at the DA market:

$$\begin{aligned} \underset{x,y}{\text{minimize}} \quad & c(x) + p \int_0^y (y - w) f(w) \mathrm{d}w \\ \text{subject to} \quad & x + y - D = 0 \\ & x, y \geq 0 \end{aligned}$$

which gives the optimal dispatch $x^* = y^* = 1/2$, with the DA market price $\lambda_d^* = 1$. The RT market price is $\lambda_r^* = p$ if there is any shortfall. The expected payoff of the wind farm is

$$\pi^* = \lambda_d^* y^* - p \int_0^{y^*} (y^* - w) f(w) \mathrm{d}w = 1/4.$$

Suppose that the wind farm claims that it can produce $1/4$ amount of power with probability 1. Then, the optimal dispatch is $\hat{x} = 3/4, \hat{y} = 1/4$, with the DA market price $\hat{\lambda}_d = 3/2$. The RT market price is $\hat{\lambda}_r = p$ when there is any shortfall. The expected payoff of the wind farm is

$$\hat{\pi} = \hat{\lambda}_d \hat{y} - p \int_0^{\hat{y}} (\hat{y} - w) f(w) \mathrm{d}w = 5/16 > \pi^*.$$

By misreporting the distribution, the wind farm is able to raise the DA market price so that it can be better off. Therefore, the extended LMP mechanism is not incentive compatible.

To elicit the true distribution of wind power generation, we propose an alternative mechanism following the same idea as in the SVCG mechanism.

Let (x^*, y^*, θ_d^*) be a solution to the following problem:

$$\begin{aligned} \underset{x,y,\theta_d}{\text{minimize}} \quad & \textstyle\sum_i c_i(x_i) + \mathbb{E}[Q(y, \theta_d, \hat{W})] \\ \text{subject to} \quad & x_i + y_i - D_i = \textstyle\sum_j B_{ij}(\theta_{d,i} - \theta_{d,j}) \quad \forall i \\ & B_{ij}(\theta_{d,i} - \theta_{d,j}) \leq \bar{f}_{ij} \quad \forall (i,j) \\ & x_i, y_i \geq 0 \quad \forall i. \end{aligned}$$

Let $(x^{-k}, y^{-k}, \theta_d^{-k})$ be a solution to the following problem:

$$\begin{aligned} \underset{x,y,\theta_d}{\text{minimize}} \quad & \textstyle\sum_i c_i(x_i) + \mathbb{E}[Q(y, \theta_d, \hat{W}) | \hat{W}_i = 0] \\ \text{subject to} \quad & x_i + y_i - D_i = \textstyle\sum_j B_{ij}(\theta_{d,i} - \theta_{d,j}) \quad \forall i \\ & B_{ij}(\theta_{d,i} - \theta_{d,j}) \leq \bar{f}_{ij} \quad \forall (i,j) \\ & x_i, y_i \geq 0 \quad \forall i. \end{aligned}$$

TABLE II
COST FUNCTIONS OF THERMAL UNITS

Unit	Cost (\$/h) (x: MW)
1	$0.0776x^2 + 20x$
2	$0.0100x^2 + 40x$
3	$0.2500x^2 + 20x$
4	$0.0100x^2 + 40x$
5	$0.0222x^2 + 20x$
6	$0.0100x^2 + 40x$
7	$0.0323x^2 + 20x$

TABLE III
CHARACTERISTICS OF WIND FARMS

Unit	Capacity (MW)	Mean (MW)	SD (MW)	Fitted cost ($p = 100$ \$/MWh) (\$/h) (x: MW)
1	101.1	37.02	33.06	$0.4933x^2 + 13.82x$
2	101.0	33.01	29.66	$0.5156x^2 + 13.84x$
3	112.0	38.57	33.08	$0.4670x^2 + 11.69x$
4	100.1	38.86	31.66	$0.5122x^2 + 8.35x$
5	206.4	74.71	65.39	$0.2480x^2 + 11.03x$
6	138.4	50.96	43.53	$0.3616x^2 + 14.09x$
7	99.8	36.15	30.65	$0.5169x^2 + 11.43x$

Let the DA payment scheme be

$$t_{d,k}(\hat{\sigma}) = \sum\nolimits_i (c_i(x_i^*) - c_i(x_i^{-k})) \quad (12)$$

and the RT payment scheme be

$$\begin{aligned} t_{r,k}(\hat{\sigma}, w_k) = \mathbb{E}[Q(y^{-k}, \theta_d^{-k}, \hat{W}) | \hat{W}_k = 0] \\ - \mathbb{E}[Q(y^*, \theta_d^*, \hat{W}) | \hat{W}_k = w_k]. \end{aligned} \quad (13)$$

Theorem 6: The mechanism specified by (12)–(13) is efficient, dominant strategy incentive compatible, and individual rational.

Note that in the ISO's problem, the DA payment is determined by the costs of the thermal units, instead of a contract price chosen by the aggregator in the aggregator's problem.

V. CASE STUDY

In this section, we focus on the ISO's problem and present a case study based on the IEEE 57-bus system [17].

There are 57 buses, 7 thermal units, and 42 loads in the system. For the cost functions of the thermal units, we use the data from MATPOWER [18], which is listed in Table II.

Consider that there is a wind farm colocated with each thermal unit. We obtain wind power generation data of seven wind farms from the NREL dataset [19], and calculate the mean and standard deviation (SD) for each wind farm. Assume that each W_i is a truncated normal random variable, which is bounded below by zero and bounded above by its capacity. Let the cost of load shedding be $p = 100$ \$/MWh. We use the Monte Carlo method to compute the cost $C_i(x_i) = \mathbb{E}[p(x_i - W_i)_+]$ with varied x_i, and fit a quadratic function to the data. The characteristics of the wind farms are listed in Table III.

First, we compare the expected social costs under two market architectures with increasing numbers of wind farms, as shown in Fig. 2. When the number of wind farms is i, it means that wind farms 1 through i are present. In either architecture, the expected social cost decreases as the number of wind farms

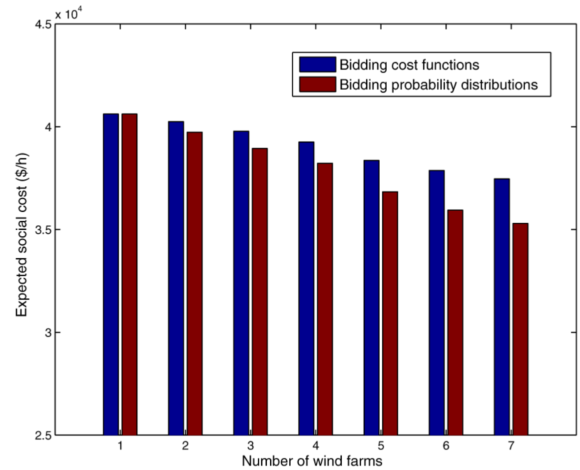

Fig. 2. Expected social cost versus the number of wind farms under two market architectures.

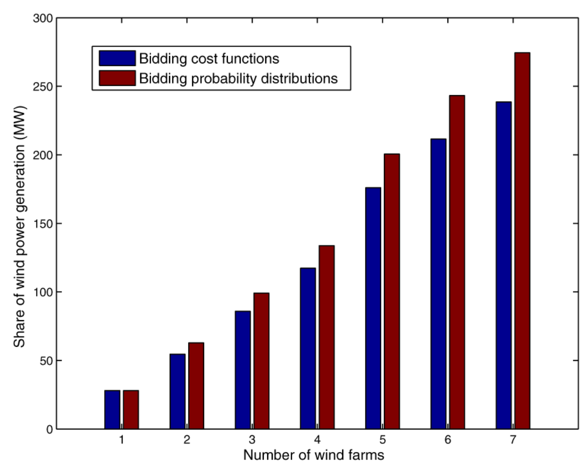

Fig. 3. Share of wind power generation versus the number of wind farms under two market architectures.

increases. For any fixed number of wind farms, the proposed architecture always achieves a lower expected social cost than the conventional architecture.

Next, we compare the share of wind power generation under two market architectures, as shown in Fig. 3. In either architecture, the share of wind power generation increases as the number of wind farms increases. For any fixed number of wind farms, the share in the proposed architecture is higher than that in the conventional architecture, due to the value of aggregation.

We then compare the expected payoff of wind farm 1 under two market architectures, as shown in Fig. 4. The payoff of wind farm i in the conventional architecture is defined the same as that of a thermal unit: $\pi_i = \lambda_i x_i - C_i(x_i)$, where λ_i is the LMP. As the number of wind farms increases, the expected payoff of wind farm 1 in the conventional architecture decreases slightly, since there is increasing competition, yet the wind power penetration is not that high. The payoff of wind farm i in the proposed architecture is given by (12) and (13). When there are other wind farms, the expected payoff of wind farm 1 is evidently higher than when it is the only wind farm, again due to the value of aggregation. For any fixed number

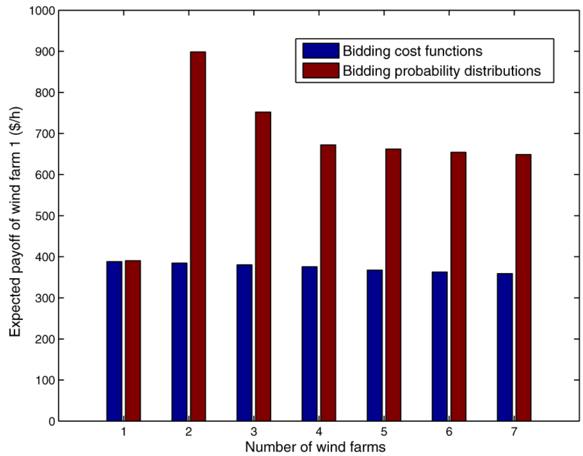

Fig. 4. Expected payoff of wind farm 1 versus the number of wind farms under two market architectures.

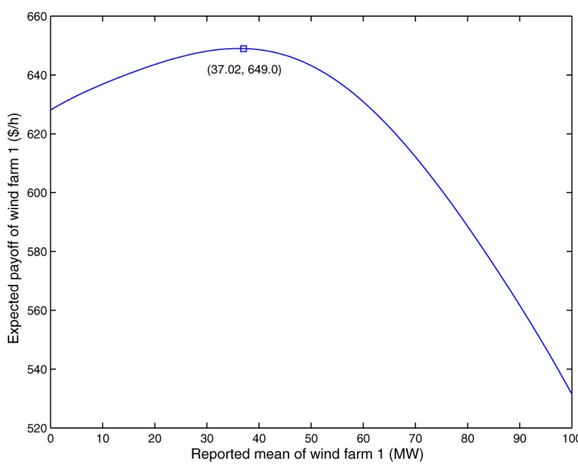

Fig. 5. Expected payoff of wind farm 1 versus the reported mean in the proposed architecture.

of wind farms, the expected payoff in the proposed architecture is always higher than that in the conventional architecture. Intuitively, the surplus is the information rent that the ISO has to pay to extract the true information.

Lastly, we demonstrate that the proposed mechanism specified by (12) and (13) is incentive compatible. Consider the case in which all the seven wind farms are present, and focus on wind farm 1. Assume that the capacity and the SD of generation are truthfully revealed. The only parameter wind farm 1 can misreport is the mean of generation. Fig. 5 depicts the expected payoff of wind farm 1 as the reported mean varies. The maximum value 649.0 $\$/\text{h}$ is attained at 37.02 MW, which is exactly the actual mean of W_1. Therefore, truth-telling is the best strategy of wind farm 1. On the contrary, the LMP mechanism is not incentive compatible, in which agents may be better off by strategically misreporting its private information.

VI. Conclusion

Motivated by the observation that it is more efficient for wind power producers to bid probability distributions of generation than bid cost functions, we propose the stochastic resource auction paradigm, which is aligned with the existing two-settlement system.

In the aggregator's problem, we propose two incentive compatible mechanisms. We then make several generalizations: general objective function, selecting multiple providers, and inclusion of fixed cost. In the ISO's problem, we present a counterexample to show that the LMP-based mechanisms are subject to manipulation. We propose an alternative mechanism which is incentive compatible and thus induces the stochastic economic dispatch. The presented case study illustrates the advantages of the proposed architecture.

To implement the proposed architecture, certain regulatory changes are necessary. In the future, we will also explore the connection between truthful elicitation and profit maximization, and adapt the current results for more practical settings.

The presented work, along with the proposed future work, will potentially provide economic solutions for integrating renewables into the smart grid network.

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