Dynamic Economic Dispatch Game: The Value of Storage

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***Abstract*—We formulate a dynamic economic dispatch game in which each generator has its own electricity storage device. The operation of storage introduces time-coupling constraints. We focus on how the use of storage may affect the market structure and market outcomes in the locational marginal pricing mechanism. We first show that even when the independent system operation is unaware of the storage, there exists an efficient bid profile that induces the optimal dispatch. We then demonstrate that the use of storage does not increase the room for strategic play, but may improve the equilibrium outcomes. We provide sufficient conditions under which there exist efficient Nash equilibria. Furthermore, we propose the marginal contribution pricing mechanism that guarantees efficient outcomes.**

***Index Terms*—Dynamic economic dispatch, energy storage system, game theory, locational marginal pricing, smart grid.**

I. Introduction

INCREASING penetration of renewable energy sources like wind and solar introduces various new challenges due to their high variability and volatility. Thus, there is lots of effort in developing efficient energy storage systems that can scale in industrial sizes. Once large-scale storage systems are widely available, they are likely to be deployed by generators (and even distributors and consumers) to manage volatility in generation and prices. From the independent system operator's (ISO's) perspective, how should economic dispatch be done with storage? Does the ISO need to dispatch both generation and storage? Does storage increase the room for strategic play already available to generators? This paper attempts to address these questions.

Consider a day-ahead market, where the ISO conducts economic dispatch over a 24-hour horizon. Such a problem is modeled as dynamic economic dispatch (DED), and various models have been proposed in the literature [1]–[4]. In those models, decisions at different periods are typically coupled by ramping constraints; the fuel cost function may not be smooth due to valve-point effects [5]; there can be prohibited operating zone constraints which make the feasible region nonconvex. Hence, most work on DED focuses on developing efficient algorithms that solve for the optimal dispatch [6]–[9]. The recent DED models have incorporated the uncertainty of wind power generation, and solution methods in such stochastic settings have been studied [10], [11].

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We study the DED problem from a widely different point of view. First, we propose a DED model in which each generator has its own electricity storage device. The operation of storage introduces time-coupling constraints, so that the instant supply (power injected into the network) may differ from the instant generation. We note that a lot of recent work has modeled the use of storage in various contexts [12]–[14]. Second, we consider a game-theoretic setting in which generators are strategic agents. We investigate the equilibrium outcomes in the locational marginal pricing (LMP) mechanism [15].

We focus on how the use of storage may affect the market structure and market outcomes. To implement the optimal dispatch, there are two major challenges. First, the ISO is unaware of the storage, or has no access to the control of the storage. Given the lack of information, is it possible for the ISO to conduct the optimal dispatch? Second, generators may not have incentives to reveal their true cost functions. While the use of storage may reduce the cost on the supply side, it is unclear whether it would introduce more opportunities for the generators to manipulate the market.

Our contributions are the following. We first show that there exists an efficient bid profile which internalizes the operation of storage, and hence induces the optimal dispatch. The implication is that the current market structure does not have to change: the ISO makes supply decisions only, and the generators make generation and storage decisions. Storage can be treated as generation in some periods, and as load in others. We then demonstrate that the use of storage does not expand the room for strategic play. On the contrary, the use of storage may sustain a Nash equilibrium that would otherwise not exist; it may also improve the inefficiency of the worst equilibrium. We provide sufficient conditions under which there exist efficient Nash equilibria. Furthermore, as an alternative to the LMP mechanism, we propose the marginal contribution pricing (MCP) mechanism which always induces an efficient Nash equilibrium.

The paper is organized as follows. In Section II, we introduce the DED model and formulate the DED game. In Section III, we construct an efficient bid profile to address the issue of storage-unawareness. In Section IV, we investigate how the use of storage affects the equilibrium outcomes in the LMP mechanism. As an alternative, we propose the

MCP mechanism in Section V. In Section VI, we present a case study to complement the analysis in the previous sections. Section VII concludes the paper.

II. Problem Statement

A. Model

Consider a connected power network consisting of I buses and N generators. The set of generators at each bus i is denoted by $\mathcal{N}_i$. Let T be the total number of periods in the planning horizon. The demand at each bus i and each period t is inelastic, denoted by $D_{i,t} \geq 0$.

Each generator n has an electricity storage device, with a storage capacity $C_n \geq 0$. Let $x_{n,t}$ be the supply (power injected into the network) at period t, $z_{n,t}$ be the generation at period t, and $y_{n,t}$ be the state of charge at the end of period t. Then the storage dynamics is given by

$$y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t}, \ \ \forall t,$$

with $y_{n,0} = 0$ for all n. Let the cost function of generator n at period t be $c_{n,t} : \mathbb{R}_+ \to \mathbb{R}_+ \cup \{+\infty\}$, so that the cost of producing $z_{n,t}$ is $c_{n,t}(z_{n,t})$. The cost function is assumed to be increasing, convex, and differentiable on its effective domain $\{z_{n,t} \geq 0 : c_{n,t}(z_{n,t}) < +\infty\}$. Note that generation limits have been incorporated in the cost functions by allowing $c_{n,t}$ to be $+\infty$.

We adopt a DC power flow model [16] as usual in the context of economic dispatch, in which each branch *i-j* is characterized by B_{ij}, the (i, j)-th element of the susceptance matrix. Let $\theta_{i,t}$ be the voltage phase angle at bus i and period t. Then the active power flow over branch *i-j* is given by

$$f_{ij,t} = B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big).$$

The bus power balance equation for bus i and period t is the following:

$$\sum_{n\in N_i} x_{n,t} - D_{i,t} = \sum_{j} f_{ij,t}.$$

Let $f_{ij}^{\max}$ be the flow limit of branch *i-j* such that $f_{ij}^{\max} = f_{ji}^{\max} \geq 0$. The branch power flow constraint for branch *i-j* and period t is the following:

$$f_{ij,t} \leq f_{ij}^{\max}.$$

The system problem is to determine an optimal schedule of supply and generation that minimizes the social cost while satisfying the unit and transmission constraints. Formally, it is the following convex optimization problem:

$$\min \ \sum_{t=1}^{T}\sum_{n=1}^{N} c_{n,t}(z_{n,t}) \tag{1a}$$

$$\text{s.t.} \ \sum_{n\in\mathcal{N}_i} x_{n,t} - D_{i,t} = \sum_{j} B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big), \quad \forall i, \forall t, \tag{1b}$$

$$B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big) \leq f_{ij}^{\max}, \quad \forall (i,j), \forall t, \tag{1c}$$

$$y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t}, \quad \forall n, \forall t, \tag{1d}$$

$$y_{n,t} \leq C_n, \quad \forall n, \forall t, \tag{1e}$$

$$x_{n,t}, y_{n,t}, z_{n,t} \geq 0, \quad \forall n, \forall t. \tag{1f}$$

TABLE I
NOMENCLATURE

I	number of buses
N	number of generators
$\mathcal{N}_i$	set of generators at bus i
T	number of periods
$D_{i,t}$	demand at bus i and period t
C_n	storage capacity of generator n
$x_{n,t}$	supply of generator n at period t
$y_{n,t}$	state of charge of generator n at the end of period t
$z_{n,t}$	generation of generator n at period t
$c_{n,t}$	cost function of generator n at period t
B_{ij}	(i,j)-th element of the bus susceptance matrix
$\theta_{i,t}$	voltage phase angle at bus i and period t
$f_{ij}^{\max}$	flow limit of branch *i-j*
$\lambda_{i,t}$	dual associated with the bus power balance equation
$\mu_{ij,t}$	dual associated with the branch power flow constraint
$\nu_{n,t}$	dual associated with the storage dynamics
$\xi_{n,t}$	dual associated with the storage capacity constraint
$b_{n,t}$	bid of generator n at period t
M	some large enough constant
$(\cdot)^{**}$	solution to the system problem (1)
$(\cdot)^{*}$	solution to the storage-unaware DED problem (2)–(3)
$\hat{(\cdot)}$	solution to the storage-unaware DED game (4)–(5)

For each t, since $\sum_i(\sum_{n\in\mathcal{N}_i} x_{n,t} - D_{i,t}) = \sum_i\sum_j B_{ij}(\theta_{i,t} - \theta_{j,t}) = 0$, the system of linear equations (1b) over i is underdetermined with respect to $\{\theta_{i,t}\}$. In fact, only the phase angle differences matter. Thus, for computational purposes, one may choose bus 1 as the slack bus by setting $\theta_{1,t} = 0$ for all t. Since the N generators have been started up so that the fixed cost does not affect the dispatch, we further assume that $c_{n,t}(0) = 0$ for all n and for all t without loss of generality.

For future purposes, associate the dual variable $\lambda_{i,t}$ with (1b), $\mu_{ij,t}$ with (1c), $\nu_{n,t}$ with (1d), and $\xi_{n,t}$ with (1e). Throughout the paper, we assume that the system problem (1) is always feasible. Denote the primal and dual optimal solution by $\{x_{n,t}^{**}, y_{n,t}^{**}, z_{n,t}^{**}, \theta_{i,t}^{**}, \lambda_{i,t}^{**}, \mu_{ij,t}^{**}, \nu_{n,t}^{**}, \xi_{n,t}^{**}\}$, which we call the optimal dispatch.

We summarize the notations in Table I. To implement the optimal dispatch, there are two major challenges: the issue of storage-unawareness, and the issue of incentive compatibility. We first formulate the storage-unaware DED problem that considers the former, and then propose the storage-unaware DED game that incorporates the latter as well.

Remark 1: The proposed DED model has certain limitations. Since the focus of this paper is the role of storage, we do not consider ramping constraints and contingency constraints. A comprehensive DED model is left for future work.

B. Storage-Unaware DED Problem

Since the ISO is unaware of the storage devices, it solves the following problem:

$$\min \ \sum_{t=1}^{T}\sum_{n=1}^{N} c_{n,t}(x_{n,t}) \tag{2a}$$

$$\text{s.t.} \ \sum_{n\in\mathcal{N}_i} x_{n,t} - D_{i,t} = \sum_{j} B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big), \quad \forall i, \forall t, \tag{2b}$$

$$B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big) \leq f_{ij}^{\max}, \quad \forall (i,j), \forall t, \tag{2c}$$

$$x_{n,t} \geq 0, \quad \forall n, \forall t. \tag{2d}$$

In fact, problem (2) essentially decomposes into T independent static economic dispatch (SED) problems. Denote the solution to problem (2) by $\{x^*_{n,t}, \theta^*_{i,t}\}$.

Given the supply decisions $\{x^*_{n,t}\}$, each generator n minimizes its generation cost by utilizing the storage device:

$$\min \quad \sum_{t=1}^{T} c_{n,t}(z_{n,t}) \tag{3a}$$
$$\text{s. t.} \quad y_{n,t} = y_{n,t-1} + z_{n,t} - x^*_{n,t}, \quad \forall t, \tag{3b}$$
$$y_{n,t} \le C_n, \quad \forall t, \tag{3c}$$
$$y_{n,t}, z_{n,t} \ge 0, \quad \forall t. \tag{3d}$$

Denote the solution to problem (3) by $\{y^*_{n,t}, z^*_{n,t}\}$.

We refer to problem (2)–(3) as the storage-unaware DED problem. It can be seen that its solution, $\{x^*_{n,t}, y^*_{n,t}, z^*_{n,t}, \theta^*_{i,t}\}$, is a feasible but suboptimal solution to the system problem (1).

C. Storage-Unaware DED Game

Since the generators are strategic agents, they may not have incentives to reveal their true cost functions. They are often not even able to do so, given the reported cost function constrained to be low-dimensional. Therefore, the ISO may not have the right information to conduct the optimal dispatch. To study the market outcomes with strategic generators, we reformulate the storage-unaware DED problem (2)–(3) as a game.

In the storage-unaware DED game, each generator n submits a reported cost function $b_{n,t}(\cdot)$, or bid, for each period t. In practice, the bid is typically a piecewise linear function with increasing slopes. For example, the California ISO uses 10-segment piecewise linear bids [17]. In this paper, for simplicity, we consider a two-segment piecewise linear bid parametrized by a four-dimensional signal $(r^-_{n,t}, s^-_{n,t}, r^+_{n,t}, s^+_{n,t})$, where $0 \le r^-_{n,t} \le r^+_{n,t}$, $0 \le s^-_{n,t} \le s^+_{n,t}$:

$$b_{n,t}(x_{n,t}) = \begin{cases} r^-_{n,t} x_{n,t}, & x_{n,t} \in [0, s^-_{n,t}], \\ r^-_{n,t} s^-_{n,t} + r^+_{n,t}\big(x_{n,t} - s^-_{n,t}\big), & x_{n,t} \in \big(s^-_{n,t}, s^+_{n,t}\big]. \end{cases}$$

The true cost function $c_{n,t}(\cdot)$, the bid $\bar{b}_{n,t}(\cdot)$ used in practice, and the simplified bid $b_{n,t}(\cdot)$ used in this paper are illustrated in Fig. 1. Note that we intentionally express the bid as a function of the supply, since the generation is hidden from the ISO.

Given the bids for each generator and each period, the ISO solves the following problem:

$$\min \quad \sum_{t=1}^{T} \sum_{n=1}^{N} b_{n,t}(x_{n,t}) \tag{4a}$$
$$\text{s.t.} \quad \sum_{n \in \mathcal{N}_i} x_{n,t} - D_{i,t} = \sum_j B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big), \forall i, \forall t, \tag{4b}$$
$$B_{ij}\big(\theta_{i,t} - \theta_{j,t}\big) \le f^{\max}_{ij}, \forall (i,j), \forall t, \tag{4c}$$
$$x_{n,t} \ge 0, \forall n, \forall t. \tag{4d}$$

Associate the dual variable $\lambda_{i,t}$ with (4b), and $\mu_{ij,t}$ with (4c). Denote the primal and dual optimal solution to problem (4) by $\{\hat{x}_{n,t}, \hat{\theta}_{i,t}, \hat{\lambda}_{i,t}, \hat{\mu}_{ij,t}\}$. Then $\hat{\lambda}_{i,t}$ gives the LMP at bus i and period t.

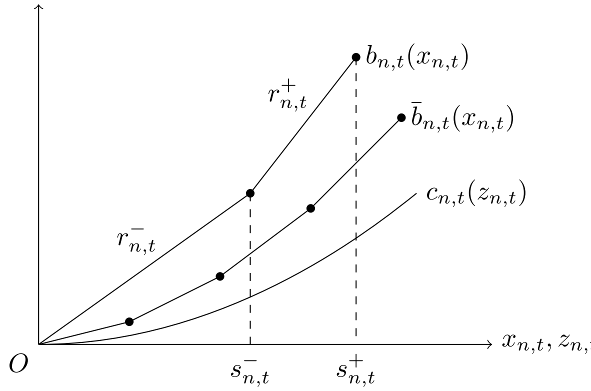

Fig. 1. True cost function $c_{n,t}(\cdot)$, the bid $\bar{b}_{n,t}(\cdot)$ used in practice, and the simplified bid $b_{n,t}(\cdot)$ used in this paper specified by a four-dimensional signal $(r^-_{n,t}, s^-_{n,t}, r^+_{n,t}, s^+_{n,t})$.

Given the supply decisions $\{\hat{x}_{n,t}\}$, each generator n solves the same problem as (3):

$$\min \quad \sum_{t=1}^{T} c_{n,t}(z_{n,t}) \tag{5a}$$
$$\text{s.t.} \quad y_{n,t} = y_{n,t-1} + z_{n,t} - \hat{x}_{n,t}, \forall t, \tag{5b}$$
$$y_{n,t} \le C_n, \forall t, \tag{5c}$$
$$y_{n,t}, z_{n,t} \ge 0, \forall t. \tag{5d}$$

Associate the dual variable $\nu_{n,t}$ with (5b), and $\xi_{n,t}$ with (5c). Denote the primal and dual optimal solution to problem (5) by $\{\hat{y}_{n,t}, \hat{z}_{n,t}, \hat{\nu}_{n,t}, \hat{\xi}_{n,t}\}$.

The payoff of generator n located at bus i is given by

$$\pi_n = \sum_{t=1}^{T} \Big(\hat{\lambda}_{i,t}\hat{x}_{n,t} - c_{n,t}\big(\hat{z}_{n,t}\big)\Big).$$

Thus, given a bid profile $\{b_{n,t}(\cdot)\}$, the solution to (4)–(5) specifies the outcome of the storage-unaware DED game. If no generator can get a higher payoff by a unilateral deviation, then the bid profile is called a Nash equilibrium. Further, if the outcome coincides with the optimal dispatch (as a solution to (1)), then the bid profile is called an efficient Nash equilibrium.

Remark 2: It turns out that a two-dimensional bid space consisting of one-segment linear bids is rich enough for the constructive proofs in the main analysis. Yet we retain two-segment linear bids for the sake of generality of the results.

III. Efficient Bids: Resolving the Issue of Storage-Unawareness

In this section, we address the first challenge. We show that there exists a bid profile which induces the optimal dispatch, despite the fact that the ISO is unaware of the storage devices.

The idea is that the bids can internalize the effect of the operation of the storage, so that the optimal dispatch, as a solution to the system problem (1), is also a solution to the storage-unaware DED game (4)–(5).

Proposition 1: Let M be a large enough constant. For each generator n located at bus i, let

$$b_{n,t}(x_{n,t}) = \lambda^{**}_{i,t} x_{n,t}, \;\; 0 \le x_{n,t} \le M, \;\; \forall t. \tag{6}$$

Then the solution to the system problem (1) is also a solution to the storage-unaware DED game (4)–(5).

Proof: It is clear that strong duality holds in problem (1), (4) and (5). Therefore, the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality [18].

The solution to the system problem (1), $\{x^{**}_{n,t}, y^{**}_{n,t}, z^{**}_{n,t}, \theta^{**}_{i,t}, \lambda^{**}_{i,t}, \mu^{**}_{ij,t}, \nu^{**}_{n,t}, \xi^{**}_{n,t}\}$, satisfies the following KKT conditions:

$$\left(\nu^{**}_{n,t} - \lambda^{**}_{i,t}\right)x^{**}_{n,t} = 0, \quad \forall (n,i), \forall t, \quad \text{(7a)}$$

$$\nu^{**}_{n,t} - \lambda^{**}_{i,t} \geq 0, \quad \forall (n,i), \forall t, \quad \text{(7b)}$$

$$\sum_j B_{ij}\left(\lambda^{**}_{i,t} - \lambda^{**}_{j,t} + \mu^{**}_{ij,t} - \mu^{**}_{ji,t}\right) = 0, \quad \forall i, \forall t, \quad \text{(7c)}$$

$$\sum_{n \in \mathcal{N}_i} x^{**}_{n,t} - D_{i,t} - \sum_j B_{ij}\left(\theta^{**}_{i,t} - \theta^{**}_{j,t}\right) = 0, \quad \forall i, \forall t, \quad \text{(7d)}$$

$$\mu^{**}_{ij,t}\left(B_{ij}\left(\theta^{**}_{i,t} - \theta^{**}_{j,t}\right) - f^{\max}_{ij}\right) = 0, \quad \forall (i,j), \forall t, \quad \text{(7e)}$$

$$B_{ij}\left(\theta^{**}_{i,t} - \theta^{**}_{j,t}\right) - f^{\max}_{ij} \leq 0, \quad \forall (i,j), \forall t, \quad \text{(7f)}$$

$$x^{**}_{n,t} \geq 0, \quad \forall n, \forall t, \quad \text{(7g)}$$

$$\mu^{**}_{ij,t} \geq 0, \quad \forall (i,j), \forall t, \quad \text{(7h)}$$

$$\left(\nu^{**}_{n,t} - \nu^{**}_{n,t+1} + \xi^{**}_{n,t}\right)y^{**}_{n,t} = 0, \quad \forall n, \forall t, \quad \text{(7i)}$$

$$\nu^{**}_{n,t} - \nu^{**}_{n,t+1} + \xi^{**}_{n,t} \geq 0, \quad \forall n, \forall t, \quad \text{(7j)}$$

$$\left(c'_{n,t}\left(z^{**}_{n,t}\right) - \nu^{**}_{n,t}\right)z^{**}_{n,t} = 0, \quad \forall n, \forall t, \quad \text{(7k)}$$

$$c'_{n,t}\left(z^{**}_{n,t}\right) - \nu^{**}_{n,t} \geq 0, \quad \forall n, \forall t, \quad \text{(7l)}$$

$$y^{**}_{n,t} - y^{**}_{n,t-1} - z^{**}_{n,t} + x^{**}_{n,t} = 0, \quad \forall n, \forall t, \quad \text{(7m)}$$

$$\xi^{**}_{n,t}\left(y^{**}_{n,t} - C_n\right) = 0, \quad \forall n, \forall t, \quad \text{(7n)}$$

$$y^{**}_{n,t} - C_n \leq 0, \quad \forall n, \forall t, \quad \text{(7o)}$$

$$y^{**}_{n,t}, z^{**}_{n,t}, \xi^{**}_{n,t} \geq 0, \quad \forall n, \forall t, \quad \text{(7p)}$$

where $\nu^{**}_{n,T+1} = 0$ for all n.

It can be seen that given the bid profile as in (6), $\{x^{**}_{n,t}, \theta^{**}_{i,t}, \lambda^{**}_{i,t}, \mu^{**}_{ij,t}\}$ satisfies the KKT conditions of (4). Moreover, given $\{x^{**}_{n,t}\}$, $\{y^{**}_{n,t}, z^{**}_{n,t}, \nu^{**}_{n,t}, \xi^{**}_{n,t}\}$ satisfies the KKT conditions of (5). Hence, the solution to (1) is also a solution to (4)–(5). ■

IV. Equilibrium Analysis: The Value of Storage

In this section, we present examples to show that the use of storage may improve the equilibrium outcomes. Moreover, we provide sufficient conditions under which there exist efficient Nash equilibria.

A. Improving Equilibrium Outcomes

The first example shows that the use of storage may sustain a Nash equilibrium that would otherwise not exist.

Example 1: Consider the network as shown in Fig. 2. The network has two buses, with generator 1 at bus 1 and generator 2 at bus 2. The branch flow limit is $f^{\max} = \infty$. The planning horizon has two periods, i.e., $T = 2$. There is no demand at bus 1. The demand at bus 2 is $D_1 = 10$, $D_2 = 20$. The cost functions are $c_{1,t}(z_{1,t}) = z_{1,t}, z_{1,t} \leq 16$ and $c_{2,t}(z_{2,t}) = 2z_{2,t}$, for $t = 1, 2$.

If there is no storage, there is no Nash equilibrium. The argument is straightforward as follows. Consider $t = 2$.

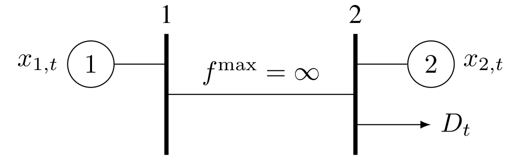

Fig. 2. Example of a two-bus, two-generator network.

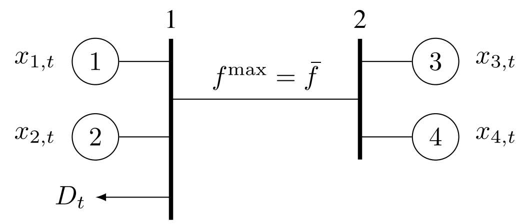

Fig. 3. Example of a two-bus, four-generator network.

Since $x_{1,2} = z_{1,2} \leq 16$ and $D_2 = 20$, it is guaranteed that $x_{2,2} = z_{2,2} \geq 4$. Then generator 2 can get an arbitrarily high payoff by bidding $b_{2,2}(x_{2,2}) = \lambda x_{2,2}$ with an arbitrarily large λ. Thus, a Nash equilibrium does not exist. In fact, the claim still holds even if there is a reserve price on the demand side.

On the other hand, a Nash equilibrium exists as long as generator 1 has its own storage with a sufficiently large capacity. Consider the following bid profile: $b_{n,t} = c_{1,t}$, for $n = 1, 2$ and $t = 1, 2$, with the outcome $x_{1,1} = 10, x_{1,2} = 20$, $z_{1,1} = 16, z_{1,2} = 14, x_2 = z_2 = \mathbf{0}$. It is easily seen that b is a efficient Nash equilibrium.

Remark 3: In this example, generator 1 has an output limit 16 for each period. If there is no storage, generator 2 has enough market power to ask for an arbitrarily high price at period 2, when the demand exceeds the output limit of generator 1. On the other hand, the use of storage makes full use of the generation capacity of generator 1 whose cost is cheaper, so that a Nash equilibrium exists.

The next example shows that the use of storage may improve the price of anarchy (PoA). The PoA is defined as the ratio between the highest cost among all Nash equilibria and the cost of the optimal dispatch.

Example 2: Consider the network as shown in Fig. 3. The network has two buses, with generator 1 and 4 at bus 1, and generator 2 and 3 at bus 2. The branch flow limit is $f^{\max} = \bar{f} < \infty$. The planning horizon has two periods, i.e., $T = 2$. The demand at bus 1 is $D_1 = D_2 = 2\bar{f}$. There is no demand at bus 2. The cost functions are

$$c_{1,1}(z_{1,1}) = z_{1,1}, \;\; c_{1,2}(z_{1,2}) = z_{1,2},$$
$$c_{2,1}(z_{2,1}) = z_{2,1}, \;\; c_{2,2}(z_{2,2}) = 2kz_{2,2},$$
$$c_{3,1}(z_{3,1}) = kz_{3,1}, \;\; c_{3,2}(z_{3,2}) = kz_{3,2},$$
$$c_{4,1}(z_{4,1}) = kz_{4,1}, \;\; c_{4,2}(z_{4,2}) = kz_{4,2}.$$

If there is no storage, we can consider two independent games for the two periods. In particular, consider the game at period 2. The optimal dispatch is

$$\left(x^{**}_{1,2}, x^{**}_{2,2}, x^{**}_{3,2}, x^{**}_{4,2}\right) = \left(2\bar{f}, 0, 0, 0\right),$$

with a total cost $2\bar{f}$. One Nash equilibrium is

$$\begin{aligned} b_{1,2}(x_{1,2}) &= 2kx_{1,2},\ x_{1,2} \le 2\bar{f}, \\ b_{2,2}(x_{2,2}) &= 2kx_{2,2},\ x_{2,2} \le 2\bar{f}, \\ b_{3,2}(x_{3,2}) &= kx_{3,2},\ x_{3,2} \le 2\bar{f}, \\ b_{4,2}(x_{4,2}) &= kx_{4,2},\ x_{4,2} \le 2\bar{f}, \end{aligned}$$

which induces the outcome

$$\left(x_{1,2}, x_{2,2}, x_{3,2}, x_{4,2}\right) = \left(\bar{f}, 0, \bar{f}, 0\right),\ (\lambda_{1,2}, \lambda_{2,2}) = (2k, k),$$

with a total cost $\bar{f} + k\bar{f}$. For example, generator 1 has no incentive to bid $\hat{b}_{1,2}(x_{1,2}) = kx_{1,2}, x_{1,2} \le 2\bar{f}$, since its new payoff, $(k-1)2\bar{f}$, would be smaller than its current payoff, $(2k-1)\bar{f}$. Thus, the price of anarchy is at least

$$\frac{\bar{f} + k\bar{f}}{2\bar{f}} = \frac{k+1}{2},$$

which is unbounded as $k \to \infty$.

On the other hand, when there is storage, all Nash equilibria are efficient. This can be easily shown by observing: (i) generator 1 has a marginal cost 1 for $t = 1, 2$; and (ii) generator 2 can also effectively have a marginal cost 1 by storing enough energy at period 1. Thus, with the use of storage, the price of anarchy is 1.

Remark 4: In this example, the cost of generator 1 is strictly the lowest at period 2. If there is no storage, it may exercise market power, which leads to an inefficient Nash equilibrium. On the other hand, the use of storage makes generator 2 essentially have the same cost as generator 1 at period 2, so that there exist only efficient Nash equilibria.

Example 3: In Example 1, if the branch flow limit $f^{\max} < D_2 = 20$, there is still no Nash equilibrium even with the use of storage. Again, this is because generator 2 can secure an amount of supply so that it has enough market power to ask for an arbitrarily high price at period 2.

In Example 2, if the storage capacity of generator 2 is smaller than D_2, it can be shown that there still exist inefficient Nash equilibria with the use of storage, due to the exercise of market power of generator 1.

Remark 5: The use of storage may not always improve the equilibrium outcomes. The storage has a capacity, or there may exist system constraints such as transmission line capacities that limit the effect of storage.

B. Existence of Efficient Nash Equilibria

We propose two sufficient conditions under either of which not only a Nash equilibrium but also an efficient one exists in the storage-unaware DED game. The idea is based on Proposition 1, where an efficient bid profile is constructed. When there is enough competition, such a bid profile can be a Nash equilibrium.

The following assumption ensures that no generator has enough market power to ask for arbitrarily high prices.

Assumption 1: The system problem (1) is still feasible if any one of the generators is excluded.

The first sufficient condition is in the following.

Definition 1 (Congestion-Free Condition): No branch power flow constraint (1c) is binding in the optimal dispatch of the system problem (1).

It is immediate to prove by construction the existence of efficient Nash equilibria under the congestion-free condition.

Theorem 1: Under Assumption 1 and the congestion-free condition, there exists an efficient Nash equilibrium in the storage-unaware DED game.

Proof: Consider the bid profile as in (6). By Proposition 1, the bid profile is efficient, which induces the optimal dispatch $\{x^{**}_{n,t}, y^{**}_{n,t}, z^{**}_{n,t}, \theta^{**}_{i,t}, \lambda^{**}_{i,t}, \mu^{**}_{ij,t}, \nu^{**}_{n,t}, \xi^{**}_{n,t}\}$. It remains to show that $\{b_{n,t}(\cdot)\}$ is a Nash equilibrium.

Consider generator n located at bus i. By the KKT conditions (7), it is easy to check that $\{x^{**}_{n,t}, y^{**}_{n,t}, z^{**}_{n,t}, \nu^{**}_{n,t}, \xi^{**}_{n,t}\}$ solves the following problem:

$$\begin{aligned} \max \quad & \sum_{t=1}^{T} \left(\lambda^{**}_{i,t} x_{n,t} - c_{n,t}(z_{n,t})\right) \\ \text{s.t.} \quad & y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t}, \forall t, \\ & y_{n,t} \le C_n, \forall t, \\ & x_{n,t}, y_{n,t}, z_{n,t} \ge 0, \forall t. \end{aligned}$$

Its current payoff is

$$\pi_n = \sum_{t=1}^{T} \left(\lambda^{**}_{i,t} x^{**}_{n,t} - c_{n,t}\left(z^{**}_{n,t}\right)\right).$$

Moreover, by the congestion-free condition, all the LMPs are equal at each period: $\lambda^{**}_{j,t} = \lambda^{**}_{1,t}$ for all j and for all t.

Suppose it changes its bid to $\{\hat{b}_{n,t}(\cdot)\}$, resulting in a new dispatch $\{\hat{x}_{n,t}, \hat{y}_{n,t}, \hat{z}_{n,t}, \hat{\theta}_{i,t}, \hat{\lambda}_{i,t}, \hat{\mu}_{ij,t}, \hat{\nu}_{n,t}, \hat{\xi}_{n,t}\}$. Note that the bid of any other generator $m \ne n$ is $b_{m,t}(x_{m,t}) = \lambda^{**}_{1,t} x_{m,t},\ 0 \le x_{n,t} \le M$, where M is large enough. Moreover, Assumption 1 implies that the demand can always be balanced without generator n. Hence, the new LMPs cannot increase whatever generator n bids. That is, $\hat{\lambda}_{i,t} \le \lambda^{**}_{1,t}$ for all t. Its new payoff would be

$$\begin{aligned} \hat{\pi}_n &= \sum_{t=1}^{T} \left(\hat{\lambda}_{i,t} \hat{x}_{n,t} - c_{n,t}\left(\hat{z}_{n,t}\right)\right) \\ &\le \sum_{t=1}^{T} \left(\lambda^{**}_{1,t} \hat{x}_{n,t} - c_{n,t}\left(\hat{z}_{n,t}\right)\right) \\ &\le \sum_{t=1}^{T} \left(\lambda^{**}_{1,t} x^{**}_{n,t} - c_{n,t}\left(z^{**}_{n,t}\right)\right) \\ &= \pi_n. \end{aligned}$$

Thus, it has no incentive to deviate. This proves that the constructed bid profile is a Nash equilibrium. ■

The second sufficient condition is the following.

Definition 2 (Monopoly-Free Condition): There are at least two, or no generators at each bus.

This condition is easier to use than the congestion-free condition, since we only need to know the placement of the generators. Moreover, Assumption 1 is automatically satisfied.

Theorem 2: Under the monopoly-free condition, there exists an efficient Nash equilibrium in the storage-unaware DED game.

Proof: Consider the bid profile as in (6). The proof is similar as before. The key is that under the monopoly-free condition and the given bid profile, the LMP at any bus and any period cannot increase by a unilateral deviation of a generator located at that bus. Consequently, no generator can get a higher payoff. This proves that the constructed bid profile is a Nash equilibrium. ∎

V. Marginal Contribution Pricing Mechanism

We have studied the LMP-based storage-unaware DED game. As an alternative, we propose the MCP mechanism, in which each generator is paid a single amount of its marginal contribution, instead of a unit price of the time-dependent locational marginal cost as in the LMP mechanism.

The proposed MCP mechanism is adapted from the Vickrey-Clarke-Groves (VCG) mechanism, a canonical mechanism in the mechanism design theory that implements efficient allocations in dominant strategies [19]. However, the standard VCG mechanism does not apply directly, since we require a finite-dimensional (and preferably low-dimensional) bid format while the true cost function can be infinite-dimensional.

In the MCP mechanism, we still use the realistic bid format, in which the reported cost is a two-segment piecewise linear function. The ISO's problem and the generator's problem are the same as (4)–(5). We only change the payment rule, so that the payoff of generator n is

$$\pi_n = w_n - \sum_{t=1}^{T} c_{n,t}(\hat{z}_{n,t}),$$

where w_n is the payment made to generator n. Let $\{\hat{x}_{m,t}^{-n}\}$ be the solution to problem (4) when generator n is excluded (so that $\hat{x}_{n,t}^{-n} = 0$ for all t). Then w_n is given by

$$w_n = \sum_{t=1}^{T}\sum_{m\neq n} b_{m,t}(\hat{x}_{m,t}^{-n}) - \sum_{t=1}^{T}\sum_{m\neq n} b_{m,t}(\hat{x}_{m,t}),$$

which is the (positive) externality that generator n imposes on the others by its participation.

Theorem 3: Under Assumption 1, there exists an efficient Nash equilibrium in the MCP-based storage-unaware DED game.

Proof: Consider the bid profile as in (6). By Proposition 1, the bid profile is efficient, which induces the optimal dispatch $\{x_{m,t}^{**}, y_{m,t}^{**}, z_{m,t}^{**}, \theta_{i,t}^{**}, \lambda_{i,t}^{**}, \mu_{ij,t}^{**}, \nu_{m,t}^{**}, \xi_{m,t}^{**}\}$. It remains to show that $\{b_{n,t}(\cdot)\}$ is a Nash equilibrium.

Consider generator n located at bus i. Its current payoff is

$$\pi_n = \sum_{t=1}^{T}\sum_{m\neq n}\left(b_{m,t}(x_{m,t}^{-n}) - b_{m,t}(x_{m,t}^{**})\right) - \sum_{t=1}^{T} c_{n,t}(z_{n,t}^{**}).$$

Note that Assumption 1 ensures that $x_{m,t}^{-n}$ is well-defined for all m and for all t. Suppose it changes its bid to $\{\hat{b}_{n,t}(\cdot)\}$, resulting in a new dispatch $\{\hat{x}_{m,t}, \hat{y}_{m,t}, \hat{z}_{m,t}, \hat{\theta}_{i,t}, \hat{\lambda}_{i,t}, \hat{\mu}_{ij,t}, \hat{\nu}_{m,t}, \hat{\xi}_{m,t}\}$. Its new payoff would be

$$\hat{\pi}_n = \sum_{t=1}^{T}\sum_{m\neq n}\left(b_{m,t}(\hat{x}_{m,t}^{-n}) - b_{m,t}(\hat{x}_{m,t})\right) - \sum_{t=1}^{T} c_{n,t}(\hat{z}_{n,t}).$$

So its payoff changes by

$$\begin{aligned}
\hat{\pi}_n - \pi_n &= \sum_{t=1}^{T}\sum_{m\neq n}\left(b_{m,t}(x_{m,t}^{**}) - b_{m,t}(\hat{x}_{m,t})\right) \\
&\quad + \sum_{t=1}^{T}\left(c_{n,t}(z_{n,t}^{**}) - c_{n,t}(\hat{z}_{n,t})\right) \\
&\leq \sum_{t=1}^{T}\left(b_{n,t}(\hat{x}_{n,t}) - b_{n,t}(x_{n,t}^{**})\right) \\
&\quad + \sum_{t=1}^{T}\left(c_{n,t}(z_{n,t}^{**}) - c_{n,t}(\hat{z}_{n,t})\right) \\
&= \sum_{t} \nu_{i,t}^{**}\left(\hat{x}_{n,t} - x_{n,t}^{**}\right) \\
&\quad + \sum_{t=1}^{T}\left(c_{n,t}(z_{n,t}^{**}) - c_{n,t}(\hat{z}_{n,t})\right) \\
&\leq 0.
\end{aligned}$$

The first equality follows from the fact that $\hat{x}_{m,t}^{-n} = x_{m,t}^{-n}$ for all m and for all t. The first inequality follows since

$$\sum_{t=1}^{T}\sum_{m=1}^{N} b_{m,t}(x_{m,t}^{**}) \leq \sum_{t=1}^{T}\sum_{m=1}^{N} b_{m,t}(\hat{x}_{m,t}).$$

By the KKT conditions (7), it is easy to check that $\{x_{n,t}^{**}, y_{n,t}^{**}, z_{n,t}^{**}\}$ solves the following problem:

$$\begin{aligned}
\max \quad & \sum_{t=1}^{T}\left(\nu_{i,t}^{**}x_{n,t} - c_{n,t}(z_{n,t})\right) \\
\text{s.t.} \quad & y_{n,t} = y_{n,t-1} + z_{n,t} - x_{n,t}, \forall t, \\
& y_{n,t} \leq C_n, \forall t, \\
& x_{n,t}, y_{n,t}, z_{n,t} \geq 0, \forall t,
\end{aligned}$$

so that the last inequality follows. Thus, it has no incentive to deviate. This proves that the constructed bid profile is a Nash equilibrium. ∎

It can be shown that in the constructed efficient Nash equilibrium (6), the payoff of each generator in the MCP mechanism is weakly higher than that in the LMP mechanism. In fact, for generator n located at bus i, we have

$$\begin{aligned}
\pi_n^{\mathrm{MCP}} &= \sum_{t=1}^{T}\sum_{m\neq n}\left(b_{m,t}(x_{m,t}^{-n}) - b_{m,t}(x_{m,t}^{**})\right) - \sum_{t=1}^{T} c_{n,t}(z_{n,t}^{**}) \\
&\geq \sum_{t=1}^{T} b_{n,t}(x_{n,t}^{**}) - \sum_{t=1}^{T} c_{n,t}(z_{n,t}^{**}) \\
&= \sum_{t=1}^{T} \nu_{n,t}^{**}x_{n,t}^{**} - \sum_{t=1}^{T} c_{n,t}(z_{n,t}^{**}) \\
&= \sum_{t=1}^{T} \lambda_{i,t}^{**}x_{n,t}^{**} - \sum_{t=1}^{T} c_{n,t}(z_{n,t}^{**}) \\
&= \pi_n^{\mathrm{LMP}}.
\end{aligned}$$

Moreover, when the congestion-free condition holds and $x_{n,t}^{**} > 0$ for all n and for all t, the payoffs are equal. This result

TABLE II
COST FUNCTIONS FOR THE IEEE 57-BUS SYSTEM

Generator	Bus	Cost (\$/hr) (x: MW)
1	1	$0.3879x^2 + 4x$
2	2	$0.0500x^2 + 8x$
3	3	$1.2500x^2 + 4x$
4	6	$0.0500x^2 + 8x$
5	8	$0.1111x^2 + 4x$
6	9	$0.0500x^2 + 8x$
7	12	$0.1613x^2 + 4x$

only applies to the constructed equilibrium. There can be other equilibria, efficient or not, in both mechanisms.

VI. CASE STUDY

In this section, we present numerical results for the IEEE power system test cases [20].

Consider the IEEE 57-bus system. There are 57 buses, 7 generators and 42 loads in this power network. Consider a 24-hour planning horizon, i.e., $T = 24$. Since the test case only gives the load data for a single period, we take the load D_i at bus i as the mean value and model the dynamic load as a negative sine wave:

$$D_{i,t} = D_i\big[1 - 0.5\sin(\pi t/12)\big],\ \ t = 1, 2, \ldots, 24,$$

which mimics the load in real-life scenarios. The cost functions for generation are listed in Table II. For simplicity, we assume that the cost function of the same generator is constant over time. The flow limits of the branches are the same, i.e., $f_{ij}^{\max} = f^{\max}$ for all (i, j). The storage capacities of the generators are the same, i.e., $C_n = C$ for all n.

We consider three scenarios. In the baseline scenario, there is no storage. In the second scenario, the ISO is unaware of the storage, while the generators bid true cost functions. This corresponds to the storage-unaware DED problem (2)–(3). The third scenario corresponds to the system problem (1), in which the ISO is aware of the storage. Alternatively, we can say that it corresponds to the storage-unaware DED game (4)–(5), in which the generators submit the efficient bid profile.

First, we assume that the storage capacity is sufficiently large, i.e., $C = \infty$. We compare the social costs under three scenarios as the flow limit increases, as shown in Fig. 4. In all the three scenarios, the social cost decreases as the flow limit increases. Moreover, the use of storage can reduce the social cost greatly. When the ISO is aware of the storage, the social cost can be even lower.

In the following, we assume that the flow limit is sufficiently large, i.e., $f^{\max} = \infty$. We focus on the optimal dispatch, i.e., the solution to the system problem (1).

Fig. 5 plots the social costs under three scenarios as the storage capacity increases. Again, the use of storage can reduce the social cost, and there is an improvement when the ISO is aware of the storage.

We examine the aggregate generation versus time under different storage capacities, as shown in Fig. 6. As the storage capacity increases, the generation curve becomes flatter, given the cost functions to be constant over time. In the case where the storage capacity is zero, the aggregate generation curve is identical to the aggregate load curve.

Since there is no congestion in the power network, the LMPs are equal for any given time. Fig. 7 plots the LMPs versus

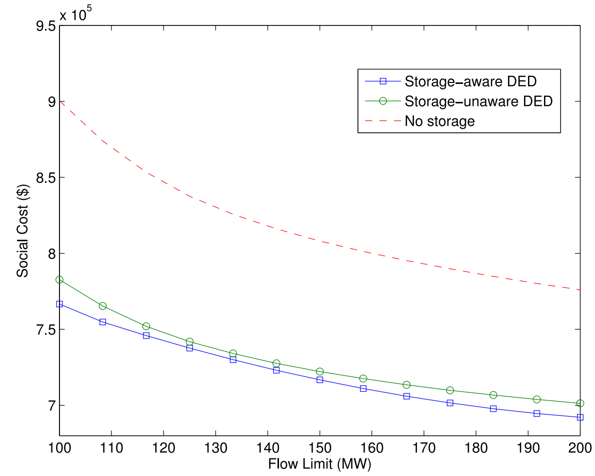

Fig. 4. Social cost versus flow limit under three scenarios, when the storage capacity is sufficiently large.

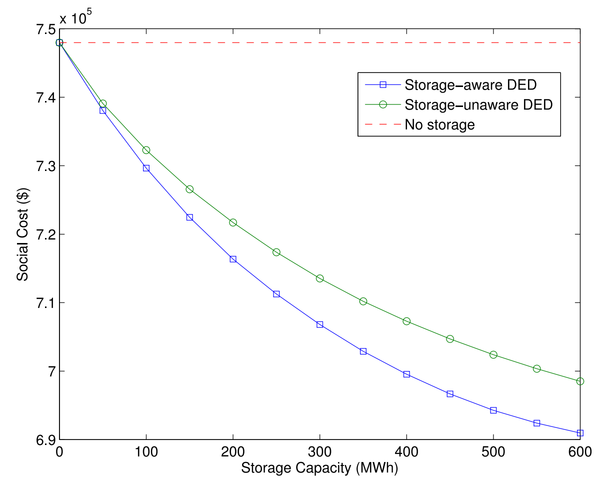

Fig. 5. Social cost versus storage capacity under three scenarios, when the flow limit is sufficiently large.

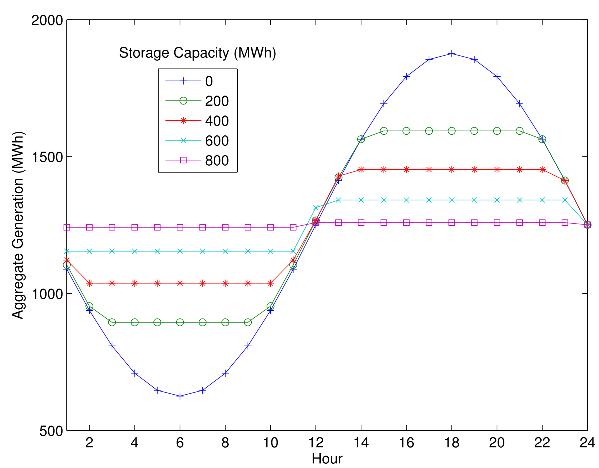

Fig. 6. Aggregate generation versus time under different storage capacities, when the flow limit is sufficiently large.

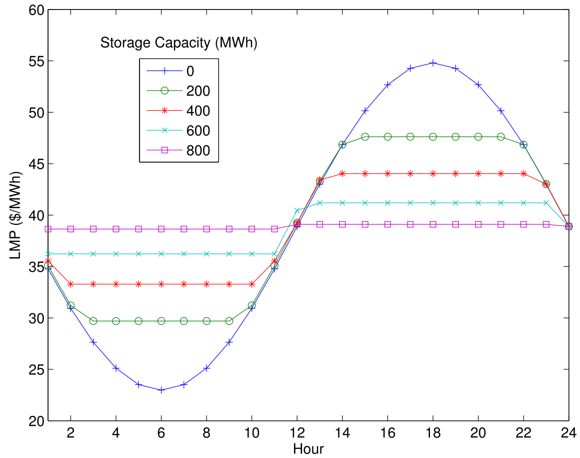

Fig. 7. LMP versus time under different storage capacities, when the flow limit is sufficiently large.

TABLE III
SOCIAL COSTS BETWEEN SED AND DED

Case	SED ($)	DED ($)	% Difference
14-bus	7.485×10^4	7.178×10^4	4.11
30-bus	1.733×10^4	1.567×10^4	9.59
57-bus	7.480×10^5	6.883×10^5	7.98
118-bus	1.379×10^6	1.310×10^6	4.97

time under different storage capacities. As the storage capacity increases, the LMP curve becomes flatter. That is, the use of storage may reduce the price volatility.

Lastly, we compare the social costs between the SED approach (the dispatch in the baseline scenario) and the DED approach (the optimal dispatch) under different test cases, when the flow limit and the storage capacity are sufficiently large. From the results listed in Table III, we can see that overall, the DED approach reduces the social cost.

VII. CONCLUSION

We formulate a DED model in which each generator has its own electricity storage device. The operation of storage introduces time-coupling constraints. To implement the optimal dispatch, there are two major challenges: the issue of storage-unawareness, and the issue of incentive compatibility.

To address the first issue, we construct an efficient bid profile that internalizes the operation of storage, and hence induces the optimal dispatch. This implies that the current market structure does not have to change. As for the second issue, we demonstrate that the use of storage does not expand the room for strategic play, but may improve the equilibrium outcomes: it may sustain a Nash equilibrium that would otherwise not exist; it may also improve the price of anarchy. While the use of storage does not guarantee the existence nor the efficiency of Nash equilibria, we provide two sufficient conditions under either of which not only a Nash equilibrium but also an efficient one exists. Furthermore, as an alternative to the LMP mechanism, we propose the MCP mechanism which always induces an efficient Nash equilibrium.

To focus on the value of storage, and the strategic behavior of the generators, we use a simplified model to provide the insights into the DED problem. One can apply the same idea to a more comprehensive model. In future work, we will investigate whether it is possible to adapt the two sufficient conditions for a larger class of scenarios, and generalize the results to the stochastic setting.

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