Pricing Sequential Forward Power Contracts

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Abstract—**Pricing mechanisms for multistage power procurement under uncertainty are studied in this paper. We first consider the most natural pricing mechanism in which the power at each stage is priced uniformly at the temporal marginal price. Under the truth-telling assumption, we show that this mechanism is price equivalent, efficient and flexibility subsidizing. Motivated by the observation that this mechanism is vulnerable to misreporting, we propose an alternative mechanism termed dynamic marginal contribution pricing, for which periodic ex post incentive compatibility is proved. Case studies illustrate the theoretical results obtained in this paper.**

I. Introduction

In a wholesale electricity market, the system operator (SO) clears a series of markets for the balancing of power at each instant. To absorb the uncertainty of supply and demand at the real time, the current practice is allocating enough reserve capacity, and conducting load shedding if necessary. With the increasing penetration of renewables (e.g., considering California's ambitious goal of 33% by 2020), that approach will no longer be cost effective, since substantial reserve capacity will be needed.

A growing consensus in designing operational schemes for the grid with deepening penetration of renewables lies in the need of explicit modeling of the uncertainty. Differing in the criteria chosen to optimize, stochastic programming and robust optimization are among the often recommended methods for operation of the grid under uncertainty. Optimizing expected cost criteria (similar to stochastic programming), risk-limiting dispatch (RLD) [1] implicitly hedges against the risk of not meeting operating constraints, through sequential optimal energy purchases, thus eliminating the need for separate reserve markets based on deterministic factors of reliability. In particular, RLD acknowledges the fact that future decisions based on more accurate forecasts may correct current decisions, in contrast with the decoupled dispatch in the current practice.

In this paper, we formulate a multistage stochastic control problem, in which the SO procures power sequentially from generators with varied lead times, employing the up-to-date information at each stage. As in the RLD setting, the optimal dispatch policy can be identified by dynamic programming. Meanwhile, the SO needs to set a pricing mechanism that supports the optimal dispatch policy. In other words, the generators should be reimbursed properly for their costs so that they would like to participate in the market in an efficient manner. Thus, the focus of this paper is to design incentivizing pricing mechanisms that facilitate efficient participation of the generators in the dynamic setting.

Most wholesale electricity markets employ marginal pricing or uniform pricing as the settlement scheme, in which all generators are paid the same market-clearing price [2]. As an extension, locational marginal pricing (LMP) reflects how this marginal price may vary at different buses when transmission congestion and losses are taken into account [3]. The LMP mechanism is used independently in each forward market, aligned with the decoupled dispatch as in the current practice. Based on the two-stage stochastic programming approach, pricing mechanisms that account for uncertainty in energy-only markets have been studied in [4], [5].

We propose a generalized marginal pricing mechanism which reflects the effect of generation flexibility and information refinement as time advances, hence referred to as temporal marginal pricing (TMP). The TMP mechanism has quite a few appealing properties, but suffers from untruthful reporting of the strategic generators. In particular, a fast-start generator may pretend to be a slow-start generator so as to secure a higher profit. Generators may also change their bids from stage to stage in the dynamic setting. For example, a generator does not have to participate in the next forward market even if it claims to be flexible enough in the current market. To address the issue of incentive compatibility, we propose the dynamic marginal contribution pricing (DMCP) mechanism, based on the dynamic mechanism design framework developed recently [6]. In the DMCP mechanism, truth-telling is a best response regardless of the history and the current state of the other agents, provided that all the other agents report truthfully.

The paper is organized as follows. In Section II, we model the multistage energy procurement problem as a multistage stochastic control problem. In Section III, we propose the TMP mechanism and derive its properties. In Section IV, we propose the DMCP mechanism that resolves the issue of incentive compatibility. We present case studies in Section V to demonstrate the performance of the proposed mechanisms. Section VI concludes the paper.

II. Problem Statement

Consider a fixed delivery time. Let L be the load and W be the wind power generation, both of which are random variables. To meet the net demand $D = L - W$, the SO procures power sequentially at $T - 1$ forward markets, and make any

necessary adjustments at the real-time spot market. We refer to the t-th forward market as stage t (where $t = 1, \ldots, T-1$), and the spot market as stage T. Denote the measurements at stage t by y_t, which provides information about L and W. The measurements may include current load, wind power generation, and weather forecasts. The information available at stage t is then $Y_t = \{y_s, s \le t\}$. Since L and W are realized at stage T, Y_T contains the realization of D.

There are N generators with varied lead times, specified by the commitment deadline $\tau_i \in \{1, \ldots, T-1\}$ for generator i. At each stage t before the commitment deadline (where $t = 1, \ldots, \tau_i$), generator i is scheduled a quantity $x_{i,t} \ge 0$ to be delivered at the real time, so that its total generation is $x_i = \sum_{t=1}^{\tau_i} x_{i,t}$. Note that $x_{i,t}$ should be based on the current information Y_t, but not the future information. The cost function $c_i(x_i)$ of generator i is assumed to be convex, non-decreasing and differentiable. At stage T when D is realized, the SO makes a recourse decision $x_0 \in \mathbb{R}$ to balance the supply and demand. The recourse cost function $c_0(x_0)$ is assumed to be convex, non-increasing on $\mathbb{R}_-$, and non-decreasing on $\mathbb{R}_+$. The model of multistage energy procurement is shown in Fig. 1.

The system problem is to find a dispatch policy $\{\phi_0(\cdot), \phi_{i,t}(\cdot), t \le \tau_i\}$ that minimizes the social cost:

$$\text{minimize} \quad \mathbb{E}\left[\sum_{i=1}^{N} c_i\left(\sum_{t=1}^{\tau_i} x_{i,t}\right) + c_0(x_0)\right] \quad (1a)$$

$$\text{subject to} \quad \sum_{i=1}^{N}\sum_{t=1}^{\tau_i} x_{i,t} + x_0 = D, \quad (1b)$$

$$x_{i,t} \ge 0, \ \forall(i,t): t \le \tau_i, \quad (1c)$$

$$x_{i,t} = \phi_{i,t}(Y_t), \ \forall(i,t): t \le \tau_i, \quad (1d)$$

$$x_0 = \phi_0(Y_T). \quad (1e)$$

It can be easily shown that it is always optimal not to schedule $x_{i,t} > 0$ until stage τ_i for generator i. Therefore, we obtain the following problem which is equivalent to (1):

$$\text{minimize} \quad \mathbb{E}\left[\sum_{i=0}^{N} c_i(x_i)\right] \quad (2a)$$

$$\text{subject to} \quad \sum_{i=0}^{N} x_i = D, \quad (2b)$$

$$x_i \ge 0, \ i = 1, \ldots, N, \quad (2c)$$

$$x_i = \phi_i(Y_{\tau_i}), \ i = 1, \ldots, N, \quad (2d)$$

$$x_0 = \phi_0(Y_T). \quad (2e)$$

The profit of generator i is given by

$$\pi_i(x_i, p_i) = p_i - c_i(x_i),$$

where p_i is the payment made to generator i. The focus of this paper is to design proper payment schemes that facilitate efficient participation of the generators.
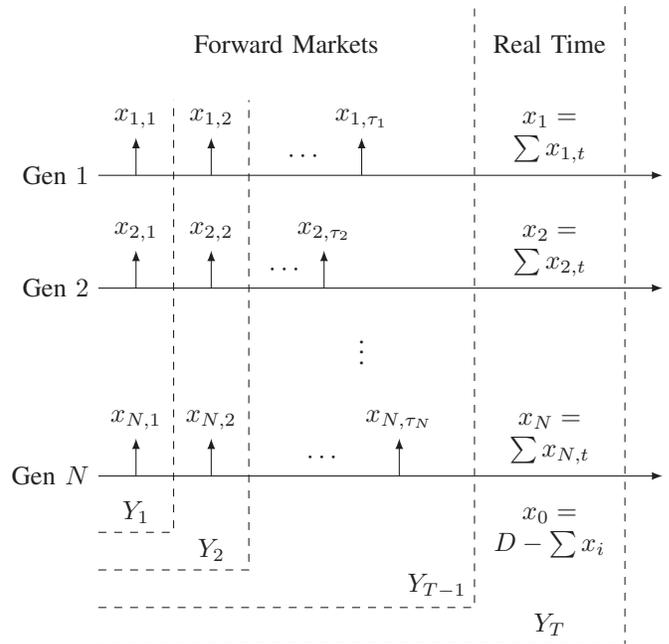

Fig. 1. Multistage energy procurement.

III. Temporal Marginal Pricing

Problem (2) can be written in the following form:

$$\text{minimize} \quad \mathbb{E}\left[\sum_{i=0}^{N} c_i(x_i)\right] \quad (3a)$$

$$\text{subject to} \quad \sum_{i:\tau_i=1} x_i = z_1, \quad :\lambda_1 \quad (3b)$$

$$z_1 + \sum_{i:\tau_i=2} x_i = z_2, \quad :\lambda_2 \quad (3c)$$

$$\vdots$$

$$z_{T-2} + \sum_{i:\tau_i=T-1} x_i = z_{T-1}, \quad :\lambda_{T-1} \quad (3d)$$

$$z_{T-1} + x_0 = D, \quad :\lambda_T \quad (3e)$$

$$x_i \ge 0, \ i = 1, \ldots, N, \quad (3f)$$

$$x_i = \phi_i(Y_{\tau_i}), \ i = 1, \ldots, N, \quad (3g)$$

$$x_0 = \phi_0(Y_T), \quad (3h)$$

where λ_t's are the dual variables associated with the constraints weighted by the corresponding probabilities. The purpose of this transform is twofold. First, problem (3) prompts a solution method based on dynamic programming, where z_t (the total scheduled quantity by stage t) along with Y_{t+1} can be viewed as the state at stage $t+1$. Second, λ_t, as a function of Y_t, has an economic meaning: it is the cost of procuring a marginal increment of power at stage t. Hence, we define as λ_t the temporal marginal price (TMP) at stage t. Accordingly, the induced pricing mechanism is referred to as the TMP mechanism, where $p_i = \lambda_{\tau_i} x_i$. The profit of generator i is

$$\pi_i(x_i, \lambda_{\tau_i}) = \lambda_{\tau_i} x_i - c_i(x_i).$$

Example 1: Let $T = 3$ (e.g., representing day-ahead, 15-minute and 5-minute real-time markets). At stage 1, D is a random variable such that $Pr(D = 100) = Pr(D = 120) = 0.5$. At stage 2, the forecast of D is exact. Let $N = 2$, $\tau_i = i$, and $c_i(x_i) = x_i^2$ for $i = 1, 2$. Let $c_0(x_0) = 500 \max\{x_0, 0\}$, the interpretation of which is that the excessive generation is spilled for free, and that the value of lost load is 500. The system problem is the following:

$$
\begin{aligned}
\text{minimize} \quad & c_1(x_1) + 0.5(c_2(x_2^l) + c_0(x_0^l)) \\
& + 0.5(c_2(x_2^h) + c_0(x_0^h)) \\
\text{subject to} \quad & 0.5(x_1 + x_2^l + x_0^l) = 0.5 \times 100, \\
& 0.5(x_1 + x_2^h + x_0^h) = 0.5 \times 120, \\
& x_1, x_2^l, x_2^h \geq 0,
\end{aligned}
$$

where l (low) denotes the event $\{D = 100\}$, and h (high) denotes the event $\{D = 120\}$. The solution gives the optimal dispatch and the TMPs:

$$
x_1 = 55, \ x_2^l = 45, \ x_2^h = 65, \ x_0^l = x_0^h = 0,
$$
$$
\lambda_1 = 110, \ \lambda_2^l = \lambda_3^l = 90, \ \lambda_2^h = \lambda_3^h = 130.
$$

We will revisit this example later.

A. Properties

First, we show that given the current information, the TMPs at all future stages are the same in expectation.

Proposition 1 (Price Equivalence): For any r, s, t such that $r \leq s \leq t$,

$$
\mathbb{E}[\lambda_s | Y_r] = \mathbb{E}[\lambda_t | Y_r].
$$

Proof: Consider the Karush-Kuhn-Tucker (KKT) conditions of problem (3). For z_{T-1}, we have

$$
\frac{\partial}{\partial z_{T-1}} (\lambda_{T-1}(-z_{T-1}) + \mathbb{E}[\lambda_T z_{T-1} | Y_{T-1}]) = 0,
$$

or $\lambda_{T-1} = \mathbb{E}[\lambda_T | Y_{T-1}]$. Similarly, we have

$$
\lambda_t = \mathbb{E}[\lambda_{t+1} | Y_t], \ t = 1, \ldots, T-1.
$$

By the tower property of conditional expectation, we obtain

$$
\mathbb{E}[\lambda_s | Y_r] = \mathbb{E}[\mathbb{E}[\lambda_{s+1} | Y_s] | Y_r] = \mathbb{E}[\lambda_{s+1} | Y_r] = \cdots = \mathbb{E}[\lambda_t | Y_r].
$$

∎

The preceding result implies that there is no intertemporal arbitrage opportunity in expectation. Second, we show that the TMP mechanism is efficient in a competitive environment: given the TMPs, the optimal dispatch maximizes each generator's profit.

Proposition 2 (Efficiency): Let $(x_0^*, x_1^*, \ldots, x_N^*)$ be the optimal dispatch and $(\lambda_1^*, \ldots, \lambda_T^*)$ be the TMPs. For all i,

$$
x_i^* \in \arg\max_{x_i \geq 0} \lambda_{\tau_i}^* x_i - c_i(x_i).
$$

Proof: From the KKT conditions of (3), we have, for x_i:

$$
\begin{aligned}
(c_i'(x_i^*) - \lambda_{\tau_i}^*) x_i^* &= 0, \\
c_i'(x_i^*) - \lambda_{\tau_i}^* &\geq 0, \\
x_i^* &\geq 0,
\end{aligned}
$$

which coincides with the KKT conditions of the individual's problem displayed above. Hence, the result follows. ∎

Third, we show that the TMP mechanism acknowledges the contribution of flexibility: for any two generators with the same cost functions, the more flexible one always secures a weakly higher profit in expectation.

Proposition 3 (Flexibility Subsidization): For any i, j, t such that $c_i(\cdot) = c_j(\cdot) = c(\cdot)$ and $t \leq \tau_i \leq \tau_j$,

$$
\mathbb{E}[\pi_i | Y_t] \leq \mathbb{E}[\pi_j | Y_t].
$$

Proof: Define $\varphi(\lambda) = \max_x \lambda x - c(x)$, which is convex, since $\lambda x - c(x)$ is linear in λ and pointwise maximum preserves convexity. Therefore,

$$
\pi_i = \varphi(\lambda_{\tau_i}^*) = \varphi(\mathbb{E}[\lambda_{\tau_j}^* | Y_{\tau_i}]) \leq \mathbb{E}[\varphi(\lambda_{\tau_j}^*) | Y_{\tau_i}] = \mathbb{E}[\pi_j | Y_{\tau_i}],
$$

where the first and the last equalities follows from Proposition 2, the second equality follows from Proposition 1, and the inequality follows from Jensen's inequality. Taking conditional expectation on both sides, we obtain

$$
\mathbb{E}[\pi_i | Y_t] \leq \mathbb{E}[\mathbb{E}[\pi_j | Y_{\tau_i}] | Y_t] = \mathbb{E}[\pi_j | Y_t].
$$

∎

B. Vulnerability to Misreporting

The preceding properties are based on the underlying assumption that the characteristics of the generators are truthfully revealed. Since the generators are strategic agents, they may not have incentives to reveal such private information, as illustrated in the following example.

Example 2: Consider Example 1. We show that even if the cost functions are truthfully revealed, there is still plenty of room to game the system.

Suppose generator 2 claims that its lead time is the same as generator 1, so that it should also be dispatched at stage 1. Then the SO solves the following problem:

$$
\begin{aligned}
\text{minimize} \quad & c_1(x_1) + c_2(x_2) + 0.5c_0(x_0^l) + 0.5c_0(x_0^h) \\
\text{subject to} \quad & 0.5(x_1 + x_2 + x_0^l) = 0.5 \times 100, \\
& 0.5(x_1 + x_2 + x_0^h) = 0.5 \times 120, \\
& x_1, x_2 \geq 0.
\end{aligned}
$$

The solution gives the dispatch and the TMPs:

$$
x_1 = x_2 = 60, \ x_0^l = -20, \ x_0^h = 0,
$$
$$
\lambda_1 = 120, \ \lambda_2^l = \lambda_3^l = 0, \ \lambda_2^h = \lambda_3^h = 240.
$$

In this case, the profit of generator 2 is 3600. When it reports truthfully, the expected profit is 3125. Thus, generator 2 has an incentive to misreport its lead time.

Such misreporting has serious consequences. In the short term, the resulting dispatch may not be optimal, and the social cost may increase. In the long term, it fails to provide adequate incentives for investment in flexible dispatchable resources. In the next section, we will propose an alternative mechanism that addresses the issue of incentive compatibility.

IV. DYNAMIC MARGINAL CONTRIBUTION PRICING

We adapt the recent results on dynamic mechanism design for the multistage energy procurement problem, with exogenous uncertainty. The proposed dynamic marginal contribution pricing (DMCP) mechanism pays each generator its stagewise marginal contribution at each stage, which induces truth-telling as a best response in the dynamic sense.

A. Setup

Define a generic state vector $\theta = (t, Y, z, \mathcal{P}, c, \tau)$, where t is the stage index, Y is the information set, z is the total scheduled power, $\mathcal{P}$ is the set of participating generators, c is the profile of cost functions, and τ is the profile of commitment deadlines. Given θ, we define a generic optimization problem $\mathbf{OPT}(\theta)$, whose optimal value is denoted by $C(\theta)$:

$$
\begin{aligned}
\text{minimize} \quad & \mathbb{E}\left[\sum_{j \in \mathcal{P}} c_j(x_j) + c_0(x_0) \,\Big|\, Y\right] \\
\text{subject to} \quad & \sum_{j \in \mathcal{P}} x_j + x_0 = D - z, \\
& x_j \geq 0, \ \forall j \in \mathcal{P}, \\
& x_j = \phi_j(Y_{\tau_j}), \ \forall j \in \mathcal{P}, \\
& x_0 = \phi_0(Y_T).
\end{aligned}
$$

Let $\theta_t = (t, Y_t, z_{t-1}, \mathcal{P}, c, \tau)$, where z_t is the total scheduled power by stage t, and $\mathcal{P} = \{j : \tau_j \geq t\}$. Then the social cost to go at stage t is given by $C(\theta_t)$. Moreover, let $\theta_t^{-i} = (t, Y_t, z_{t-1}, \mathcal{P}\backslash\{i\}, c, \tau)$. Then the social cost to go at stage t when generator i is excluded is given by $C(\theta_t^{-i})$. For generator i at stage t, the marginal contribution to go is

$$
M_i(\theta_t) = C(\theta_t^{-i}) - C(\theta_t),
$$

and the stagewise marginal contribution is

$$
m_i(\theta_t) = C(\theta_t^{-i}) - C(\theta_t) - \mathbb{E}[C(\theta_{t+1}^{-i}) - C(\theta_{t+1})].
$$

The idea of the mechanism is to make generator i secure the stagewise marginal contribution as the profit at each stage. Thus, the payment made to generator i at stage t is given by

$$
\begin{aligned}
& p_{i,t}(\theta_t) \\
&= C(\theta_t^{-i}) - C(\theta_t) - \mathbb{E}[C(\theta_{t+1}^{-i}) - C(\theta_{t+1})] + \mathbf{1}_{\{\tau_i=t\}} c_i(x_i) \\
&= C(\theta_t^{-i}) - \mathbb{E}[C(\theta_{t+1}^{-i})] - \sum_{j \neq i: \tau_j = t} c_j(x_j).
\end{aligned}
$$

The expected profit of generator i is then its marginal contribution to go at stage 1:

$$
\mathbb{E}[\pi_i] = \mathbb{E}\left[\sum_{t=1}^{\tau_i} p_{i,t}(\theta_t) - c_i(x_i)\right] = M_i(\theta_1).
$$

B. Mechanism

To completely specify the mechanism, we need to consider cases in which the reports are not truthful, or the generators do not follow the bids placed in the previous stages. Let $(\hat{c}_{i,t}(\cdot), \hat{\tau}_{i,t})$ be the report of generator i at stage t. Truth-telling means that $\hat{\tau}_{i,t} = \tau_i$, and $\hat{c}_{i,t}(\cdot)$ is effectively truthful:

$$
\hat{c}_{i,t}(x_{i,t}) = c_i\left(x_{i,t} + \sum_{s=1}^{t-1} x_{i,s}\right) - c_i\left(\sum_{s=1}^{t-1} x_{i,s}\right),
$$

where $x_{i,s}$ is the scheduled power of generator i at stage s, which can be positive if generator i has misreported τ_i before. We formally specify the DMCP mechanism in the following.

Algorithm 1 DMCP Mechanism

$z_0 \leftarrow 0$
for $t \leftarrow 1$ **to** $T - 1$ **do**
 Observe Y_t
 $\mathcal{P} \leftarrow \{j : \hat{\tau}_{j,t} \geq t\}$
 Receive $\hat{c}_t \leftarrow \{\hat{c}_{j,t}(\cdot)\}_{j \in \mathcal{P}}, \hat{\tau}_t \leftarrow \{\hat{\tau}_{j,t}\}_{j \in \mathcal{P}}$
 $\hat{\theta}_t \leftarrow (t, Y_t, z_{t-1}, \mathcal{P}, \hat{c}_t, \hat{\tau}_t)$
 Solve $\mathbf{OPT}(\hat{\theta}_t)$, obtain $C(\hat{\theta}_t)$, schedule $\{x_{j,t}\}_{j \in \mathcal{P}}$
 $z_t \leftarrow z_{t-1} + \sum_j x_{j,t}$
 for $i \in \mathcal{P}$ **do**
  $\hat{\theta}_t^{-i} \leftarrow (t, Y_t, z_{t-1}, \mathcal{P}\backslash\{i\}, \hat{c}_t, \hat{\tau}_t)$
  Solve $\mathbf{OPT}(\hat{\theta}_t^{-i})$, obtain $C(\hat{\theta}_t^{-i})$
  $\hat{\theta}_{t+1|t}^{-i} \leftarrow (t+1, Y_{t+1}|Y_t, z_t, \mathcal{P}\backslash\{j : \hat{\tau}_j = t\}\backslash\{i\}, \hat{c}_t, \hat{\tau}_t)$
  Solve $\mathbf{OPT}(\hat{\theta}_{t+1|t}^{-i})$, obtain $C(\hat{\theta}_{t+1|t}^{-i})$
  $p_{i,t}(\theta_t) \leftarrow C(\hat{\theta}_t^{-i}) - \mathbb{E}[C(\hat{\theta}_{t+1|t}^{-i})] - \sum_{j \neq i} \hat{c}_{j,t}(x_{j,t})$
 end for
end for
$x_0 \leftarrow D - z_{T-1}$

Example 3: Consider Example 1. At stage 1, $x_1 = 55$, $p_{1,1} = 9075$, so that the profit of generator 1 is

$$
\pi_1 = p_{1,1} - c_1(x_1) = 6050.
$$

Moreover, $p_{2,1} = -16125$, which means that generator 2 makes a payment to the system. This ensures that generator 2 is truth-telling at stage 1, and in particular, it would remain in the system. Indeed, it is anticipated that generator 2 would secure a high revenue in the next stage.

At stage 2, if $D = 100$, $x_2^l = 45$, $p_{2,2}^l = 22500$; if $D = 120$, $x_2^h = 65$, $p_{2,2}^h = 32500$. The expected profit of generator 2 is

$$
\begin{aligned}
\mathbb{E}[\pi_2] &= p_{2,1} + 0.5(p_{2,2}^l - c_2(x_2^l)) + 0.5(p_{2,2}^h - c_2(x_2^h)) \\
&= 8250.
\end{aligned}
$$

C. Properties

The DMCP mechanism is efficient, in the sense that it induces the optimal dispatch. Since the expected profit is the marginal contribution, the DMCP mechanism also subsidizes flexibility. Now we show that it is incentive compatible in the following dynamic sense.

Definition 1: A mechanism is *periodic ex post incentive compatible*, if for each agent, truth-telling is a best response

regardless of the history and the current state of the other agents, provided that all the other agents report truthfully.

Theorem 1: The DMCP mechanism is periodic ex post incentive compatible.

Proof: Let θ_t and $x_{i,t}$ be the state and dispatch given the truthful reports of all the agents. Let $\hat{\theta}_t$ and $\hat{x}_{i,t}$ be the state and dispatch given any report of generator i, and truthful reports of the other agents. By the one-stage deviation principle [7], it suffices to prove that truth-telling is a best response for each agent i at each stage t:

$$p_{i,t}(\theta_t) - c_i(x_{i,t}) + \mathbb{E}[M_i(\theta_{t+1})]$$
$$\geq p_{i,t}(\hat{\theta}_t) - c_i(\hat{x}_{i,t}) + \mathbb{E}[M_i(\hat{\theta}_{t+1|t})],$$

or

$$C(\theta_t^{-i}) - C(\theta_t) \geq C(\theta_t^{-i}) - \mathbb{E}[C(\hat{\theta}_{t+1|t}^{-i})] - \sum_{j \neq i} c_j(\hat{x}_{j,t})$$
$$- c_i(\hat{x}_{i,t}) + \mathbb{E}[M_i(\hat{\theta}_{t+1|t})],$$

or

$$C(\theta_t) \leq \sum_j c_j(\hat{x}_{j,t}) + \mathbb{E}[C(\hat{\theta}_{t+1|t})],$$

which is true since $C(\theta_t)$ is the optimal value of the social cost starting at t. ∎

The following result compares the expected profits between those two mechanisms.

Theorem 2: For each generator, the expected profit in the DMCP mechanism is weakly higher than that in the TMP mechanism assuming that truthful information is revealed.

Proof: Let π_i^D be the expected profit of generator i in the DMCP mechanism, and π_i^T be that in the TMP mechanism. We have

$$\pi_i^D = \mathbb{E}\left[\sum_{j=0}^N c_j(x_j^{-i})\right] - \mathbb{E}\left[\sum_{j=0}^N c_j(x_j)\right]$$
$$\geq \mathbb{E}\left[\sum_{j \neq i} c_j'(x_j)(x_j^{-i} - x_j) - c_i(x_i)\right]$$
$$\geq \mathbb{E}\left[c_i'(x_i)(x_i - x_i^{-i}) - c_i(x_i)\right]$$
$$= \mathbb{E}[c_i'(x_i)x_i - c_i(x_i)]$$
$$= \pi_i^T.$$
∎

Intuitively, the surplus in the generator's profit is the information rent that the system operator has to pay to extract the true information.

V. CASE STUDIES

Consider a setting in which there are two dispatch stages (for DA and RT markets) and a real-time adjustment stage ($T = 3$). The second dispatch stage is very close to the real-time adjustment stage so that these two stages have the same information set. Wind and load data from BPA for year 2014 are used for this case study [8]. The annual average of hourly
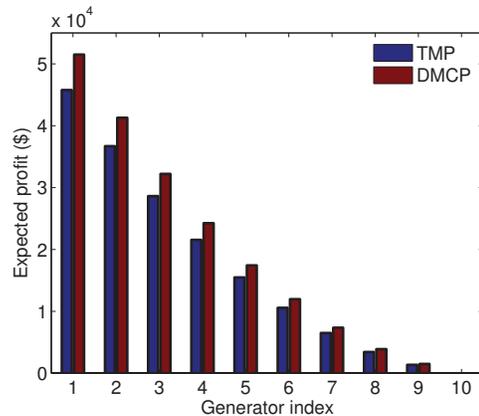

Fig. 2. Expected profit earned by each generator.

load and wind are 6281 MW and 1271 MW, respectively, with the mean wind penetration of 20.24%. The standard deviation for the forecast error is 214.1 MW, which accounts for 16.85% of the average wind generation and 3.41% of the average load.

For numerical simplicity, we fit a discrete probability distribution with 1000 outcomes using the net forecast error data, and consider a representative problem with the DA forecast being the mean net demand (average load minus average wind). Under such a setting, the information set in the first dispatch stage contains the forecast of net demand and the probability distribution of the forecast error, and the information set in the second dispatch stage and the adjustment stage contains the realization of the net forecast error (which in turn leads to the realization of the net demand).

We consider 10 generators which have the cost $c_n(x_n) = 0.05x_n^2 + c_n^L x_n$ with $c_n^L = 20, 30, \ldots, 100, 200$, for $n = 1, \ldots, 10$, respectively. The first 5 generators are DA generators which have to be dispatched at the first stage due to their lead time requirements. The remaining generators are RT generators which can ramp up or down their generation level rapidly so that they be dispatched either in the first stage or in the second stage.

We simulate both the TMP and DMCP mechanisms with the dataset. Assuming that all generators report their true lead times and cost function, both mechanisms lead to efficient allocation with the optimal cost being $\$4.11 \times 10^5$. The payments provided to generators by these two mechanisms are different and so are the profits of the generators. Fig. 2 depicts the profits of all 10 generators under TMP and DMCP mechanisms. We observe that indeed each generator's profit under DMCP is higher than that under TMP. Here the payments and profits for TMP are calculated assuming that the true lead times and cost data are reported to the SO. However, gaming opportunities exist for TMP as discussed in Section III-B.

Fig. 3 demonstrates the potential efficiency loss on the system cost in the TMP mechanism if there are RT generators misreporting their lead times, i.e., pretending that they are DA generators. As there is no incentive compatibility guarantee for
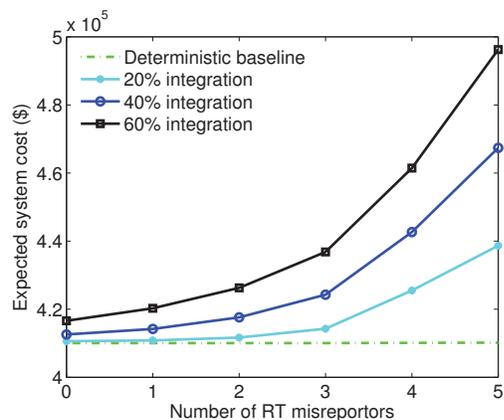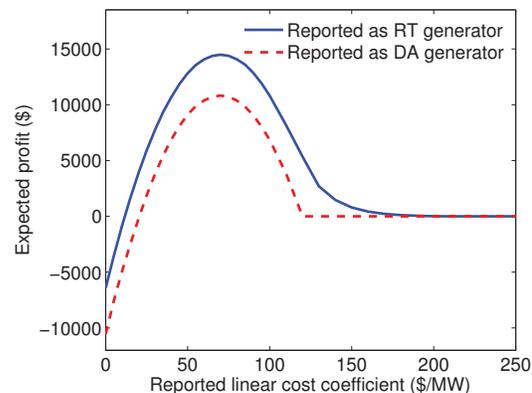
Fig. 3. System cost with misreporting in TMP.

(a) Maximum profit achieved at 70$/MW and reported as RT.

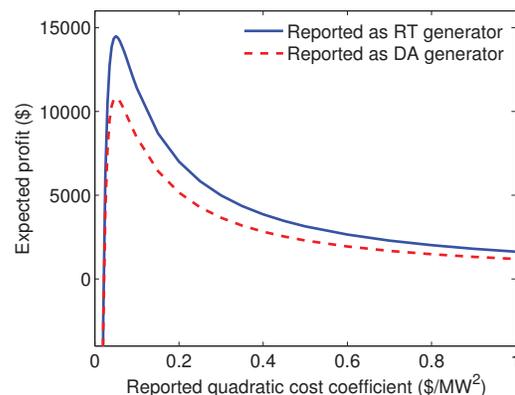(b) Maximum profit achieved at 0.05$/MW^2 and reported as RT.

Fig. 4. Generator 6's profit with misreporting in DMCP.

the TMP mechanism, this is possible in view of Example 2. Here we have performed this calculation for various forecast error levels, corresponding to no renewable generation, 20%, 40%, and 60% penetration of variable generation resources. This is done by linearly scaling up the forecast errors with error standard deviation per renewable penetration fixed to be 214.1MW/20.24% as in the current BPA data. Confirming our intuition, we see from the figure that in the case without uncertainty, misreporting of the lead times does not affect the system cost; the higher the uncertainty level in the system, the more significant the efficiency loss for the system becomes. Since we can interpret the area between the cost curves and the deterministic baseline as the *integration cost* for variable resources (cf. [9]), this suggests that misreporting leads to higher integration costs.

Finally, we illustrate numerically that DMCP is indeed incentive compatible. Fig. 4 shows the profit gained by generator 6 (with $c_6(x_6) = 0.05x_6^2 + 70x_6$) if it reports some cost function and lead time that are different from its true characteristics. The profits are evaluated using DMCP under the forecast errors with 60% penetration and assuming other generators are truth-telling. Fig. 4(a) shows that with the quadratic coefficient fixed to be its true value, the generator does not have an incentive to report a wrong lead time or a wrong linear coefficient. Similar is true for the quadratic cost coefficient if the linear coefficient is fixed (see Fig. 4(b)). In fact, it can be verified that truth-telling is the best response in the entire strategy space.

VI. CONCLUSION

We have proposed two mechanisms for the multistage energy procurement problem. The TMP mechanism generalizes the uniform pricing mechanism to the stochastic dynamic setting. With a few appealing properties, the TMP mechanism reflects the effect of generation flexibility and information refinement as time advances. However, it suffers from misreporting of the strategic generators. To address the issue of incentive compatibility, the DMCP mechanism is designed such that each generator receives the stagewise marginal

contribution as the profit at each stage. Moreover, generators would like to participate in the DMCP mechanism, where they can secure a weakly higher profit than in the TMP mechanism. The presented case studies illustrate the theoretic results.

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