1 What we learnt this week

- Basics: relationship between extensive form and normal form, behavioral strategies
- Classes of games: dynamic imperfect-information games
- Solution concepts: Nash equilibrium

2 Problems

Problem 1: Splitting 100 dollars

Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them 100 dollars in profit, but they must agree on how to split the 100 dollars. Bargaining works as follows: The two individuals each make a demand simultaneously. If their demands sum to more than 100 dollars, then they fail to agree, and each gets nothing. If their demands sum to 100 dollars or less, they do the project, each gets his demand, and the rest goes to charity. Each individual’s utility from getting $x$ dollars is $u(x) = x$. The individuals do not value charity.

(i) What are each player’s strictly dominated strategies?
(ii) What are each player’s weakly dominated strategies?
(iii) What are the pure-strategy Nash equilibria of this game?

Problem 2: Quick questions

(i) Consider an extensive-form game where each player has $N$ decision nodes. How many information sets must each player have for the game to be one of perfect information?
(ii) In a game where player $i$ has $N$ information sets indexed $n = 1, \ldots, N$ and $M_n$ possible actions at information set $n$, how many strategies does player $i$ have?

Problem 3: Extensive form versus normal form

For each of the following games, draw the normal-form representation and find the set of Nash equilibria.
Problem 4: Information sets

Which one of the following games looks like a dynamic imperfect-information game as the ones we saw in class? Which one looks “problematic”? Are all these games valid?

![Game 4.1](image1)

![Game 4.2](image2)

![Game 4.3](image3)

Problem 5: Predation game

(i) Draw the normal-form representation.
(ii) What outcomes does IWD predict?
(iii) What is the set of pure-strategy Nash equilibria?
(iv) Which prediction would you say is more reasonable? Why?
3 Answers

Problem 1

(i) If player $i$ demands $y \geq 100$, then any strategy of player $j$ with a demand of $x \geq 0$ is payoff equivalent. Therefore, there are no strictly dominated strategies.

(ii) Any strategy demanding more than 100 dollars is weakly dominated. To show this, note that if player 2 demands $y \geq 100$, then any strategy of player 1 with $x \geq 0$ is payoff equivalent. If player 2 demands $0 \leq y < 100$, then player 1 could demand $x = 100 - y$ and would obtain a payoff of $100 - y$. Demanding $x > 100$ will give player 1 a payoff of 0. Therefore, any strategy demanding more than 100 dollars is weakly dominated.

(iii) Any pair $(x, 100 - x)$ with $100 \geq x \geq 0$ is a pure-strategy Nash equilibrium of this game. There also exist equilibria of the form $(x, y)$ where $x, y \geq 100$.

Problem 2

(i) Each player must have $N$ information sets for the game to be one of perfect information. A game is one of perfect information if each information set contains a single decision node. Otherwise, it is a game of imperfect information.

(ii) Player $i$ has $M_1 \times M_2 \times M_3 \times ... \times M_n$ strategies in this game.

Problem 3

The normal-form representations are given below.

\[
\begin{array}{c|cccc}
 & Ll & Lr & Rl & Rr \\
\hline
U & 3,3 & 3,3 & 2,1 & 2,1 \\
D & 1,2 & 4,4 & 1,2 & 4,4 \\
\end{array}
\]

Game 3.1

\[
\begin{array}{c|cc}
 & L & R \\
\hline
U & 3,3 & 2,1 \\
D & 1,2 & 4,4 \\
\end{array}
\]

Game 3.2

Now consider the set of Nash equilibria for Game 3.1. We first see if there are non-rationalizable strategies that we can eliminate. Here, $Rl$ is not rational for player 2 (note that it is strictly dominated by $Lr$), so we can ignore it when looking for the Nash equilibria. Next, we find the pure-strategy Nash equilibria by underlying the payoff to player $j$’s best response to each of player $i$’s feasible strategies, as we did last time. This gives that there are three Nash equilibria in pure strategies: $(U, Ll)$, $(D, Lr)$, and $(D, Rr)$. Finally, to find the mixed-strategy Nash equilibria, note that if player 1 plays $U$ and $D$ both with positive probability, then player 2 plays $Lr$ for sure, but then player 1’s best response is to play $D$ for sure. Hence, there is no equilibrium where player 1 plays a totally mixed strategy. But there are equilibria where player 1 plays a pure strategy and player 2 plays a mixed strategy. If player 1 plays $U$ with probability one, player 2 plays $Ll$ with probability $p$ and $Lr$ with probability $(1 - p)$, where $p$ must be such that player 1’s best response is to play $U$:

\[
EU_1(U) = EU_1(D) \iff 3 \geq p + 4(1 - p) \iff p \geq \frac{1}{3}
\]
And if player 1 plays $D$ with probability one, player 2 plays $Lr$ with probability $q$ and $Rr$ with probability $(1 - q)$ where $q \in [0, 1]$.

In sum, the set of Nash equilibria in this game is $(U; pLl, (1-p)Lr)$ where $p \geq \frac{1}{3}$ and $(D; qLr, (1-q)Rr)$ where $q \in [0, 1]$.

Next, consider the set of Nash equilibria for Game 3.2. There are two pure-strategy Nash equilibria: $(U, L)$ and $(D, R)$. And, defining $s$ as the probability with which player 1 plays $U$ and $t$ as the probability with which player 2 plays $L$, there is a unique mixed-strategy Nash equilibrium given by $(s, t) = (\frac{1}{2}, \frac{1}{2})$.

**Problem 4**

Game 4.3 has the structure of a dynamic imperfect-information game as the ones we will study in this course.\footnote{To be precise, we should check that nodes contained in the same information set have the same possible actions. We cannot verify this here as the actions are not specified on the tree.} Game 4.1 is valid but “problematic,” since player 2 forgets his own previous move. This is what is called a game of imperfect recall. We will in general impose that players possess perfect recall. Loosely speaking, perfect recall means that a player does not forget what she once knew, including her own actions. Finally, Game 4.2 is not valid since, by the definition of information set, there cannot be a different number of possible actions in two nodes contained in the same information set.

**Problem 5**

(i) The normal-form representation is

<table>
<thead>
<tr>
<th></th>
<th>Accommodate</th>
<th>Fight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out/Accommodate</td>
<td>0, 2</td>
<td>0, 2</td>
</tr>
<tr>
<td>Out/Fight</td>
<td>0, 2</td>
<td>0, 2</td>
</tr>
<tr>
<td>In/Accommodate</td>
<td>3, 1</td>
<td>-2, -1</td>
</tr>
<tr>
<td>In/Fight</td>
<td>1, -2</td>
<td>-3, -1</td>
</tr>
</tbody>
</table>

(ii) We do iterative elimination of weakly dominated strategies as follows. We first eliminate In/Fight for Firm E because it is weakly (and strictly) dominated by In/Accommodate. We next eliminate Fight for Firm I as it is weakly dominated by Accommodate. Finally, we eliminate Out/Accommodate and Out/Fight for Firm E as they are dominated by In/Accommodate. Hence, IWD predicts (In/Accommodate, Accommodate).

(iii) There are three pure-strategy Nash equilibria: (Out/Accommodate, Fight), (Out/Fight, Fight), and (In/Accommodate, Accommodate).

(iv) The only outcome that looks reasonable is the outcome predicted by IWD: (In/Accommodate, Accommodate). The reason is that (Accommodate, Accommodate) is the sole Nash equilibrium in the one-shot simultaneous-move game that follows Firm E’s entry. Thus, the firms should expect to play (Accommodate, Accommodate) if firm E enters the market. But if this is the case, then firm E should choose to enter.

Another way to say this is that the Nash equilibria (Out/Accommodate, Fight) and (Out/Fight, Fight) are supported by a “noncredible threat.” Firm E decides not to enter because it believes that Firm I is going to play Fight if it enters. But since Fight is not an equilibrium strategy for Firm I once Firm E enters, any claim by the incumbent that it will fight is not credible, and thus Firm E should anticipate that Firm I will accommodate.