Navigating structural differences between abductive and deductive reasoning in proving activities

Vasiliki Laina (vlaina@berkeley.edu)
University of California,Berkeley

Abstract

Mathematicians often use exploratory, inductive and deductive reasoning when proving (Boero, 1999). Novices are reluctant to abandon the linear deductive reasoning exemplified in formal proofs (Karunakaran, 2018). Pedemonte (2007) argued that there is a structural difference between abductive reasoning and deductive and inductive proofs. With a focus on how different forms may organize and guide participation, I designed and implemented a puzzle-like proving activity. Preliminary findings suggest that the activity design supported students' engagement in practices such as combining ideas and experimentation, and afforded a movement between abductive and deductive reasoning.

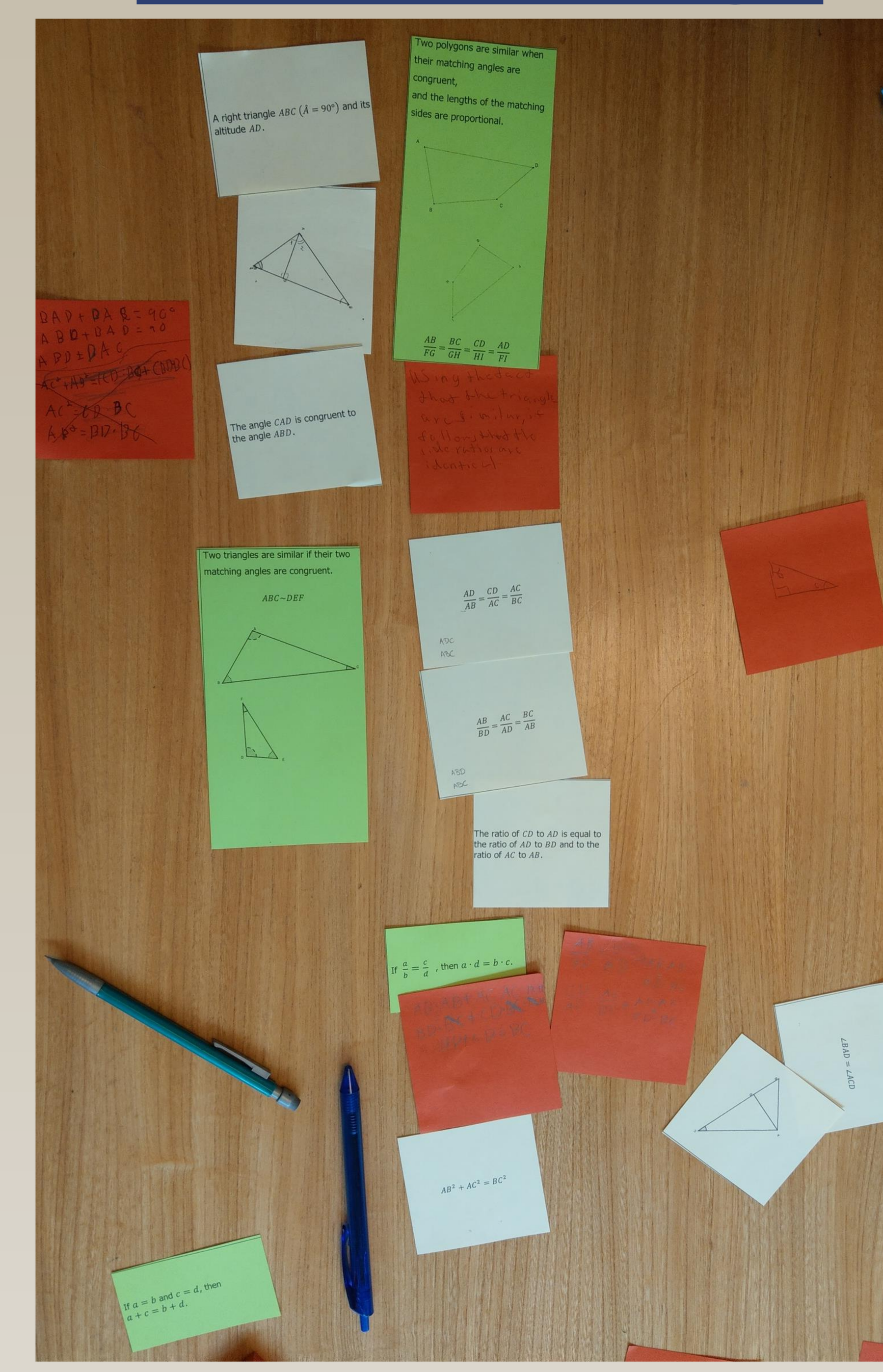
Research Question

What processes and activity design characteristics may facilitate middle school students' movement between abductive and deductive reasoning in proving activities?

Methods

- Age group: 14-18 years
- Semi-structured interviews: 2 pairs, 2 individuals
- Duration: 45-70 minutes
- Video data: activity space & participants

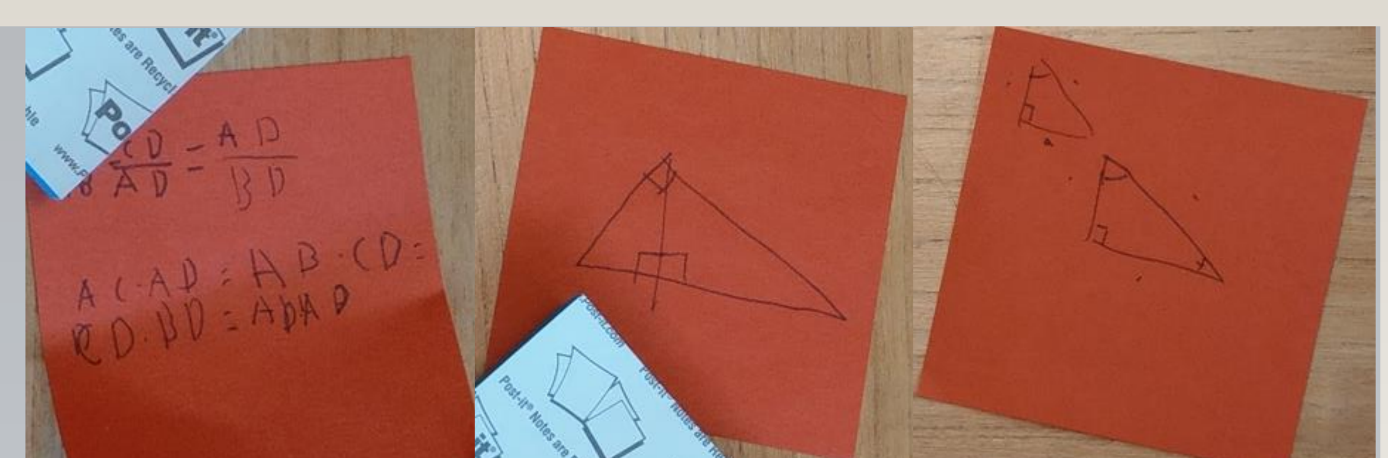
Activity Design



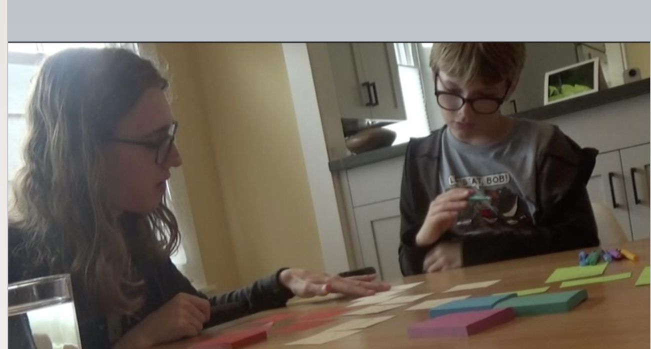
Data

8	10	9
$\frac{AB}{BD} = \frac{AC}{AD} = \frac{BC}{AB}$	$\frac{AD}{AB} = \frac{CD}{AC} = \frac{AC}{BC}$	The ratio of CD to AD is equal to the ratio of AD to BD and to the ratio of AC to AB.

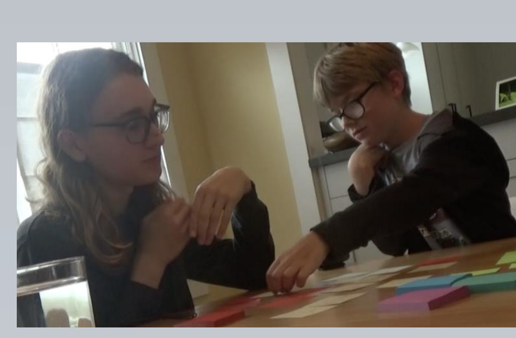
Activity cards 8, 10, and 9.

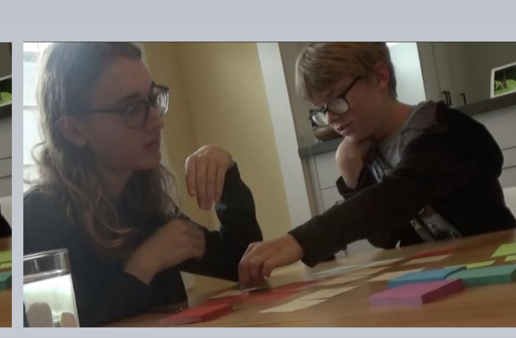


Post-its with students' notes.



Tommy places two activity cards (8,10) next to each other; drags them closer to activity card 9.





Ryan: Oh wait, Tommy. It's this one. I'll just put it there [places card 9 on top of the first post-it shown above].

Tommy: It would be nice to know that some ratios, that these [pulls card 8 closer] ratios are true [pulls activity card 10 next to activity card 8]. And, if they [refers to triangles shown on the 2nd post-it above] are congruent, then these right here are true [pointing to activity cards 8 and 10].

Findings

- **The physical proximity of the cards manifested a cognitive proximity of the information available in them.**
- **This facilitated:**
  - **experimentation with ideas,**
  - **abductive reasoning, and**
  - **transition to deductive reasoning and a final proof.**

Moving to Classrooms

- Redesign the activity to also support conjecture generation.
- Facilitating engagement across group members.
- Introduce the structure of the activity with a different context first? (e.g. Is math fun/interesting/boring? Build an argument defending your position.)

References:

Boero, P. (1999). Argumentation and mathematical proof: A complex, productive, unavoidable relationship in mathematics and mathematics education. *International Newsletter on the Teaching and Learning of Mathematical Proof*.

Karunakaran, S. S. (2018). The need for "linearity" of deductive logic: An examination of expert and novice proving processes. In A. J. Stylianides & G. Harel (Eds.), *Advances in Mathematics Education Research on Proof and Proving* (pp. 171–183). Springer International Publishing.

Pedemonte, B. (2007). How can the relationship between argumentation and proof be analysed? *Educational Studies in Mathematics*, *66*(1), 23–41.

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Berkeley
UNIVERSITY OF CALIFORNIA