Criminal Cupbearers or The Emperor(1000 wine bottle puzzle)

I have read a different version titled 'The Emperor'. My solution is developed for this particular version.But it can be studied and understood with your version titled Criminal Cupbearers.

Puzzle description: **You are the ruler of a medieval empire and you are about to have a celebration tomorrow. The celebration is the most important party you have ever hosted. You've got 1000 bottles of wine you were planning to open for the celebration, but you find out that one of them is poisoned. The poison exhibits no symptoms until death. Death occurs within twenty four hours after consuming even the minutest amount of poison. You have over a thousand slaves at your disposal and just under 24 hours to determine which single bottle is poisoned. You have a handful of prisoners about to be executed, and it would mar your celebration to have anyone else killed. What is the smallest number of prisoners you must have to drink from the bottles to be absolutely sure to find the poisoned bottle within 24 hours?**

Solution:

Assumption:
Even a drop of wine is so powerful that it doesn't get diluted when added to a bottle full of wine and the whole wine in that bottle becomes as powerful as that of the poison. The casualty, on drinking even a drop of poison occurs in second precision.

Overview:
10 Prisoners are put to test. First, they have to drink from 100 wine bottles each. For example, first Prisoner takes a mixture from 1-100 bottles. Then, he takes a mixture from bottles of corresponding 10s in each of the 10 groups of 100 bottles (10 groups are: 1-100, 101-200, 201-300…901-1000). That is, he takes a mixture of 1-10, 101-110,201-210 and so on. Finally, he takes a mixture from corresponding unit digit bottles. That is, as he is numbered 1, he takes a mixture of 1, 11, 21…101,111,121…201,211,221 and so on. Similarly all the other Prisoners also drink the respective mixtures.

By the way, each prisoner takes wine from 271 bottles.

In detail:
Out of 1000 Prisoners, select 10 agile but unfortunate Prisoners for this lethal test. Number them 1 through 10. Now ask each of them to pick any 100 wine bottles out of the 1000. First Prisoner operates 1-100; second Prisoner operates 101-200 and so on. Then, tenth Prisoner operates 901-1000.

Now, instruct all the 10 Prisoners to arrange their respective 100 bottles in the following fashion (10X10) and to prepare the mixtures of bottles of each rows and columns as illustrated below.

GROUP 1 – FIRST PRISONER – WINE BOTTLES 1-100

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	RM
R1	1	2	3	4	5	6	7	8	9	10	RM1
R2	11	12	13	14	15	16	17	18	19	20	RM2
R3	21	22	23	24	25	26	27	28	29	30	RM3
R4	31	32	33	34	35	36	37	38	39	40	RM4
R5	41	42	43	44	45	46	47	48	49	50	RM5
R6	51	52	53	54	55	56	57	58	59	60	RM6
R7	61	62	63	64	65	66	67	68	69	70	RM7
R8	71	72	73	74	75	76	77	78	79	80	RM8
R9	81	82	83	84	85	86	87	88	89	90	RM9
R10	91	92	93	94	95	96	97	98	99	100	RM10
CM	CM1	CM2	CM3	CM4	CM5	CM6	CM7	CM8	CM9	CM10	

R-Row, RM-Row Mixture, C-Column and CM-Column Mixture

GROUP 2 – SECOND PRISONER – WINE BOTTLES 101-200

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	RM
R1	1	2	3	4	5	6	7	8	9	10	RM1
R2	11	12	13	14	15	16	17	18	19	20	RM2
R3	21	22	23	24	25	26	27	28	29	30	RM3
R4	31	32	33	34	35	36	37	38	39	40	RM4
R5	41	42	43	44	45	46	47	48	49	50	RM5
R6	51	52	53	54	55	56	57	58	59	60	RM6
R7	61	62	63	64	65	66	67	68	69	70	RM7
R8	71	72	73	74	75	76	77	78	79	80	RM8
R9	81	82	83	84	85	86	87	88	89	90	RM9
R10	91	92	93	94	95	96	97	98	99	100	RM10
CM	CM1	CM2	CM3	CM4	CM5	CM6	CM7	CM8	CM9	CM10	

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GROUP 10 – TENTH PRISONER – WINE BOTTLES 901-1000

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	RM
R1	1	2	3	4	5	6	7	8	9	10	**RM1**
R2	11	12	13	14	15	16	17	18	19	20	**RM2**
R3	21	22	23	24	25	26	27	28	29	30	**RM3**
R4	31	32	33	34	35	36	37	38	39	40	**RM4**
R5	41	42	43	44	45	46	47	48	49	50	**RM5**
R6	51	52	53	54	55	56	57	58	59	60	**RM6**
R7	61	62	63	64	65	66	67	68	69	70	**RM7**
R8	71	72	73	74	75	76	77	78	79	80	**RM8**
R9	81	82	83	84	85	86	87	88	89	90	**RM9**
R10	91	92	93	94	95	96	97	98	99	100	**RM10**
CM	**CM1**	**CM2**	**CM3**	**CM4**	**CM5**	**CM6**	**CM7**	**CM8**	**CM9**	**CM10**	

Preparation of Row Mixtures (RM) and Column Mixtures (CM): The first Prisoner prepares the mixtures of 1^{st} , 2^{nd}, 3^{rd}, …9^{th} and 10^{th} rows in his group of 100 bottles(that is Group 1).

For e.g. RM2= 11+12+…19+20
Then he prepares the mixtures of 1^{st}, 2^{nd} …..9^{th} and 10^{th} columns.
For e.g. CM7= 7+17+…+87+97
All the other 9 Prisoners have performed the similar activities with their respective group of bottles at the same time.

Now, all the 10 Prisoners are asked to prepare a mixture of their respective column mixtures. (i.e., Mixture of CMs within his/her group). While preparing this, the first Prisoner is instructed to leave aside CM1 (The corresponding cell is highlighted in yellow); second Prisoner is instructed to leave aside CM2 so on.

Upon order to drink, all the Prisoners drink their mixture in hand at the same time.
The time of this drink is noted; exactly after 24 hrs (second precision), fall may occur.

Now, all the Prisoners are advised to prepare a mixture of row mixtures by walking through the other 9 groups. Accordingly, the first Prisoner has to prepare a mixture of RM1s available in all other groups. The second Prisoner has to prepare a mixture of RM2s present in all the other groups. Likewise all the 10 Prisoners prepare their respective mixture of row mixtures at the same time and are ready.

Upon order to drink, all the Prisoners drink their mixture in hand at the same time.
The time of this drink is noted; exactly after 24 hrs (second precision), fall may occur.

Now, all the Prisoners are instructed to prepare the mixture of similar column mixtures by going through all the groups from 1 to 10. Accordingly, the first Prisoner prepares a mixture of CM1s from all the 10 groups; second Prisoner prepares a mixture of CM2s and so on. Now all the 10 Prisoners prepared the mixtures and are ready.

Upon order to drink, all the Prisoners drink their mixture in hand at the same time.
The time of this drink is noted; exactly after 24 hrs (second precision), fall may occur.

The time taken to prepare the mixtures is optimum and is sufficient to distinguish the time of falls and the corresponding round of drinks. There are only 3 rounds of drinks and so there may be a maximum of 3 falls occurring in 3 slots and not more than that.

Analysis: Exactly 24 hrs after the first round drink, if suppose the first Prisoner falls, then we ascertain that the poison bottle is in the first group. That is, among 1-100. Then exactly 24hrs after the second round drink, if suppose the second Prisoner falls, then the poison bottle lies between 11-20. Again exactly 24hrs after the third drink, if suppose the third Prisoner falls, then the poison bottle is the 13^{th} one! In the above scenario, if the fall of second and third Prisoners is interchanged, then the poison bottle is the 22^{nd} one.

For any set of 3 Prisoners out of 10, we have $10C_3 = 120$ combinations. Each combination covers 6 bottles.
So, totally 120X6=720 bottles are covered.

If only 2 Prisoners fall, the first Prisoner in the first round and the second Prisoner in the second round, then the poison bottle is the 12^{th} one. If the second Prisoner falls in the third round, then the poison bottle is the 2^{nd} one. If the second Prisoner falls in the second round and the first Prisoner falls in the third round, then the poison bottle is the 11^{th} one.
For a given set of 2 Prisoners and 3 falling time slots, 6 bottles are covered. We have totally $10C_2$ Combinations.
$10C_2$=45 combinations.
So, 45X6=270 bottles are covered.

Now if only one Prisoner falls (one Prisoner falls in the third round and no one falls in the first 2 slots. This is the only possible scenario for the way the mixtures are prepared and rounds of drink administered).
Suppose the first Prisoner falls in the third slot and no falls in the first 2 slots.
Then the first bottle is the poisonous one! (Highlighted in green). No other Prisoner had it in their 3 drinks.
Similarly, if the second Prisoner alone falls, then the poison bottle is the 112^{th} one.
So ten numbers are covered in the case of only one fall as there are 10 different possibilities.

The 10 bottles are **1, 112, 223, 334, 445, 556, 667, 778, 889, 1000**(forms a series with a difference of 111. (Anything significant about this series?)

$111=10^0+10^1+10^2$.
For 3 falls, we have 720 bottles covered
For 2 falls, we have 270 bottles covered
For one fall, we have 10 bottles covered.
Totally, 720+270+10=1000 Bottles.

PS 1: For any given positive integer 'N', N^3 objects can be tested to find an odd one by employing 'N' test objects.
We also get the above series for any positive integer 'N' with a common difference of N^2+N+1.
PS 2: Geometrically, the terms in the above series represent the unit cubes in the cube 'N^3'. They are the intersections of the respective 3 planes of unit height. They form a pattern diagonally.

Thank You!