1
2
3
4

Increasing the Capacity of Signalized Intersections with Separate Left Turn Phases

7
8
9
10

Yiguang Xuan *
University of California, Berkeley
416F McLaughlin Hall #1720
University of California, Berkeley
Berkeley, CA, 94720-1720
510-642-9907
xuanyg@berkeley.edu

Carlos F. Daganzo
University of California, Berkeley
416A McLaughlin Hall #1720
University of California, Berkeley
Berkeley, CA, 94720-1720
510-642-3853
510-642-1246 (fax)
daganzo@ce.berkeley.edu

Michael J. Cassidy
University of California, Berkeley
416C McLaughlin Hall #1720
University of California, Berkeley
Berkeley, CA, 94720-1720
510-642-7702
510- 642-1246 (fax)
cassidy@ce.berkeley.edu

* Corresponding Author

38
39
40

July 31, 2010

Word count: 7,266 (5,766 + 1,500 figures and tables)

44

ABSTRACT

Separate turn phases are often used on the approach legs to intersections with heavy left turns. This wastes capacity on the approach because some of its lanes cannot discharge during its green phases. The paper shows that the problem can be eliminated by reorganizing traffic on all the lanes upstream of an intersection using a mid-block pre-signal. If drivers behave deterministically, the capacity that can be achieved is the same as if there were no left turns. However, the reorganization is so drastic that it may be counterintuitive to drivers. This can be remedied by reorganizing traffic on fewer lanes. It is shown that such partial reorganization still increases capacity significantly, even if drivers behave randomly and only one lane is reorganized. The paper shows how to optimize the design of a pre-signal system for a generic intersection. It also identifies both, the potential benefits of the proposed system for a broad class of intersections, and the domain of application where the benefits are most significant.

1. INTRODUCTION

Left turns are a significant hindrance to the smooth flow of traffic in networks involving at-grade intersections. If left turns are significant in number, separate left turn phases have to be introduced at signalized intersections to handle the flow; see (*1*). The problem with left turns is also indirectly addressed by generic optimization methods devised to determine the duration of signal phases (*2, 3*), which were later generalized to include the grouping of streams into phases (*4, 5, 6*), and more recently the lane markings (*7*).

Unfortunately, despite these optimizations, separate left turn phases reduce the fraction of time that the signal can devote to through movements, thereby reducing capacity. This reduction can increase delay if through traffic is heavy. At the network level, the reduction can mean the difference between smooth flow and gridlock.

These problems are often avoided in practice by banning and re-routing the offending left-turns. This can be done in different ways; e.g., with median U-turns, jughandles, superstreets, paired intersections, quadrant roadways, and bowties (*8, 9*). These strategies are good because they eliminate both, the left turns and the need for left turn phases, thereby increasing the intersection capacity. But the strategies also force left-turning vehicles to go through the intersection multiple times, increasing demand. This offsets some of the benefit. In addition most re-routing designs except for the paired intersection involve construction and require substantial space, which may not be available.

In view of this, the paper proposes a way of increasing capacity without banning left turns or modifying the intersection in a way that requires major construction. The focus of the analysis is a generic approach leg to a 4-way intersection controlled by a pre-timed traffic signal. The approach receives two green sub-phases: one for protected left turns only, and the other exclusively for through movements and right turns.

Currently, intersections of this type do not operate efficiently because during the protected left-turn phase, only the lanes devoted to left turns discharge. Conversely, during the through phase only the through (and right turn) lanes discharge. This is inefficient because many lanes go unused during the green phases. The problem is compounded when lanes are shared by through and left-turn vehicles, which cannot fully discharge during either sub-phase due to blocking. Imagine how much more capacity could be gained if all lanes could be made to fully discharge during both sub-phases. The paper will show how this objective can be realized by dynamically re-organizing traffic upstream of the intersection with a pre-signal.

The idea of pre-signals is not new. They have been used in Europe to give buses priority for a long time (*10*). Pre-signal systems for both, buses and bicycles have been studied (*11, 12*), respectively. Pre-signals have also been used in connection with left turns; see FIGURE 1, which illustrates the "continuous flow intersection (CFI)" concept (*13, 14*). This concept resolves the left-turn vs. opposing-through conflict at the pre-signals, as shown in the figure. Since the left-turn vehicles and opposing-through vehicles are reorganized to avoid conflicts at the intersection, they can be served in a single green phase, and this increases capacity. Interesting as it is, the CFI concept still has some drawbacks, however: (a) it requires permanent changes in layout, which cannot be turned on and off as needed; (b) it wastes lane resources unless the demands for left and through vehicles happen to be in the same balance as the

1 lanes are marked; (c) it cannot be surgically deployed on a single approach of a 4-way intersection—at
2 least 2 approaches need to be involved; and (d) it precludes bus stops from being located on the far side of
3 the intersection.

4   The proposed design is similar in spirit but overcomes these problems. Since it can be deployed
5 on a single approach leg, the single approach will be the object of analysis. Section 2 describes the
6 concept; section 3 the optimum signal settings and capacity formulae; and section 4 discusses the results,
7 including the spatial requirements of the system.
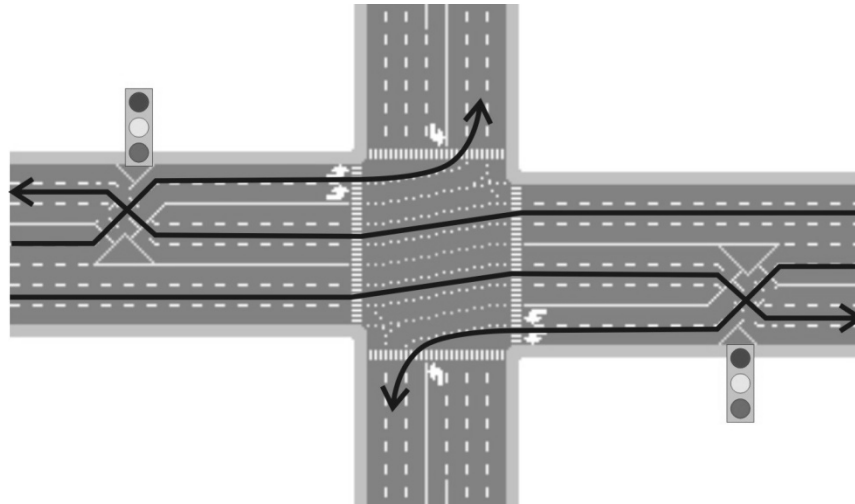
8

9 **FIGURE 1 Continuous flow intersection. Arrows show the movements allowed during the intersection's green phase for**
10 **the horizontal directions.**

11 ## 2. THE CONCEPT
12 The basic idea is depicted in FIGURE 2a. Flow is left to right. Note the pre-signal and its (dark vertical)
13 stop line. Upstream of the pre-signal lanes are marked by movement. These markings segregate left-
14 turning vehicles (LVs) and through/right-turning vehicles (TVs) onto separate sets of lanes. (The figure
15 does not show right turns for simplicity of exposition; but the system performs just the same if they are
16 included.)

17   The pre-signal runs on the same cycle as the intersection signal and alternates giving green time
18 to the two sets of lanes. Downstream of the pre-signal, variable message signs (VMSs) instruct drivers of
19 both vehicle types to use all the lanes. It may be desirable to change these instructions with traffic
20 conditions. Therefore, VMSs are preferred to lane markings because the former can be changed with time.
21 The area between the pre-signal and the signal will be called the "sorting area". This area is intended to
22 contain those transient queues created by the different offsets and phases of the signal and the pre-signal.
23 The sorting area should be long enough to ensure that these queues do not spill back to the pre-signal.

24   Here is how the system operates for one cycle if at the intersection the LV-sub-phase leads the
25 TV-sub-phase. The pre-signal starts its cycle by giving the green to the LVs while the intersection signal
26 is red. These LVs advance into the sorting area using all the lanes and wait at the intersection stop line.
27 The pre-signal then turns green for the TVs, which line up behind the LVs, again using all lanes. With this

1   arrangement, vehicles are efficiently positioned to discharge using all the lanes when the intersection
2   green phases start: first the LV-sub-phase, then the TV-sub-phase. Of course, if the LV-sub-phase lags the
3   TV-sub-phase then the order is reversed and the pre-signal starts its cycle with the TVs.

4   The duration of the pre-signal phases (for LVs and TVs) should be chosen so that the number of
5   vehicles of either type discharged by the pre-signal at saturation is close to, but does not exceed, the
6   maximum number that all the intersection lanes can jointly discharge during the corresponding
7   intersection sub-phase. In this way queues of either vehicle type clear at the signal in every sub-phase so
8   that blocking is avoided when sub-phases change. The pre-signal phases should also ensure that if there is
9   sufficient demand, then the discharged numbers should be close to the maximum. In this way both signal
10   sub-phases will be nearly saturated. This idea is illustrated in Figure 2a by the two rectangular blocks in
11   the sorting area. These blocks represent the space needed for the maximum number of vehicles that the
12   signal can discharge in its two sub-phases. The sub-phase durations shall be denoted G_L and G_T, where the
13   single-letter subscript refers to either LTs or TVs. Lengthwise, these blocks span the maximum number
14   of vehicles that can discharge from a single lane in a sub-phase. Therefore, their lengths are proportional
15   to G_L and G_T, as shown.

16
17   (a)

18
19   (b)

20
21   (c)
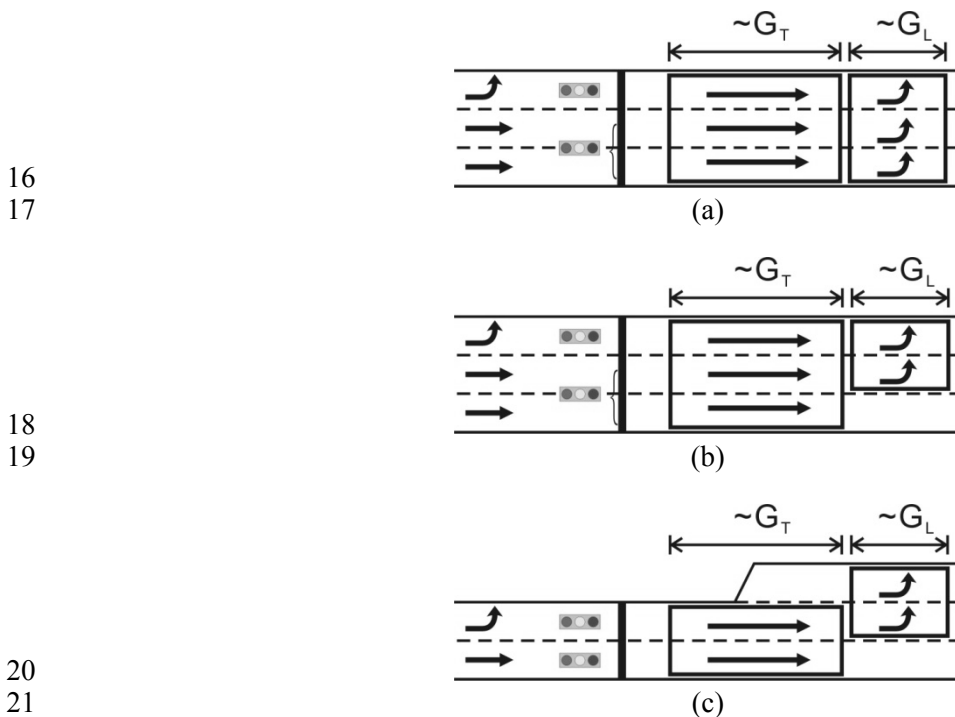
22   **FIGURE 2 The tandem intersection concept: (a) full tandem; (b) partial tandem; (c) general case. Rectangles indicate the**
23   **size of the LV and TV queues in each lane, and their order; they do not indicate the actual location of these queues.**

24   An advantage of the proposed pre-signal method when compared with approaches that require
25   fixed infrastructure and/or markings is that the phases and sub-phases of the two signals can be changed
26   with the time-of-day to be in balance with the percentage of turns. This produces (nearly) the same
27   capacity as if left turns had been banned. A disadvantage is that drivers are sometimes asked to be in a
28   counterintuitive lane; e.g., note from Figure 2a that some LVs occupy the intersection's right lane.

1   To alleviate this drawback, the full-tandem concept of FIGURE 2a can be moderated with
2   "partial-tandem" designs in which only some of the lanes in the sorting area are shared by both types of
3   vehicles; see FIGURE 2b, where only two lanes are shared. To avoid confusion with other types of shared
4   lanes, these lanes will be called "tandem lanes" from now on. The disadvantage of the partial-tandem
5   concept is that it does not engage all the lanes fully in each sub-phase. To balance user-friendliness with
6   capacity needs, one should define for any application the smallest possible set of tandem lanes that will
7   meet demand.

8   Since the number of tandem lanes can be adjusted as a decision variable, the full-tandem case can be
9   understood as a special case of partial tandem. Therefore, we will from now on focus on the latter, often
10 dropping the word "partial". The most general case (illustrated in FIGURE 2c) does not even require that
11 the number of lanes at the signal and the pre-signal be equal.

3.   CAPACITY ANALYSIS

13 The capacity of tandem and conventional intersection designs are compared here. Capacity formulae and
14 capacity-maximizing timing plans are developed, first for an idealized best case scenario in which
15 vehicles behave deterministically, and then for a more realistic scenario that captures stochastic
16 phenomena. Although the corrections introduced for the latter reduce the ideal capacity, results show that
17 the tandem design typically outperforms the conventional design nonetheless. The outperformance is
18 considerable when left turns are numerous.

19   In what follows the letters g, n, q shall denote the green times, number of lanes, and vehicle
20 discharge flows under various scenarios and at different locations. The superscript "0" will indicate a
21 conventional design without a pre-signal, and no superscript the tandem design. Subscripts "L" and "T"
22 will indicate the vehicle type, and no subscript the total. Upper and lower cases for g and n will refer to
23 the intersection and the pre-signal respectively. Note that $N_L + N_T - N$ is the number of tandem lanes,
24 which can range from 0 to N; and that $n = n_L + n_T$, since there are no shared lanes upstream of the pre-
25 signal.

26   To simplify the expressions, we shall use the cycle length as the unit for time, and the average
27 number of TVs (or LVs) that one lane can discharge in one cycle's worth of green time as the unit of TV
28 (or LV) quantity. In this system of units the green phases g_L, g_T, G_L, $G_T < 1$ are also the fractions of green,
29 and the saturation flows of both vehicle types are 1. As in Figure 2a, Figures 2b and 2c use rectangles to
30 denote the maximum number of vehicles that the signal can discharge in its two sub-phases. These
31 rectangles would have the same lengths in all the figures if the sub-phase durations were the same in the
32 three cases, but these rectangles span fewer lanes in Figures 2b and 2c. This reduced rectangular width
33 illustrates why the partial tandem system can sustain smaller flows.

34   Given as parameters are: the total green time available to the approach (G); the fraction of left-
35 turns to be accommodated ($l = q_L / q$); the original approach geometry, with no shared lanes anywhere
36 (N_L^0, N_T^0, $N^0 = N_L^0 + N_T^0$, n_L, n_T, $n = n_L + n_T$); and the modified lane definitions for the tandem design in
37 the sorting area ($N = N^0$, N_L, N_T, with $N \le N_L + N_T \le 2N$).

38   For both the conventional and tandem designs, we first solve the "optimal timing problem"; i.e.,
39 we find the maximum total flow that the approach can sustain for a given G by optimally choosing the

1   intersection sub-phase durations (G_L^0, G_T^0, G_L, and G_T), and for the tandem design the pre-signal phase
2   durations (g_L and g_T) too. In doing so, we shall assume that the lost time at the pre-signal can be neglected;
3   i.e., that the pre-signal green phases can span the whole cycle: $g_L + g_T \leq 1$. This seems reasonable as a
4   first approximation since the conflicts resolved at the pre-signal involve vehicles traveling in the same
5   direction. Likewise, we shall also assume that G is an "effective green" so we can choose any sub-phases
6   such that $G_L^0 + G_T^0 \leq G$ and $G_L + G_T \leq G$. We shall ignore the impact of this choice on the opposing flows,
7   which is reasonable if these flows are low. Otherwise, a couple of capacity constraints should be added to
8   the formulation about to be presented. The effect of high opposing flows will not be evaluated in this
9   paper for brevity.

10   With the optimum timing problem as a building block, we then solve the "optimal design problem".
11 For this problem, the lane assignments are no longer be treated as parameters, but as decision variables.

3.1. Deterministic Analysis

13 It is assumed in this section that drivers discharge from the traffic signals with deterministic headways.
14 The timing problem is presented first.

3.1.1   Optimal Timing

16 The maximum flow of the conventional design is given by the following simple linear program with the
17 green sub-phase durations as decision variables, which shall be called "CLP" (conventional linear
18 program):

19   (CLP) max q^0

20                          $q_L^0 = q^0 l$; $q_T^0 = q^0(1-l)$ (1.1)

21                              $G_L^0 + G_T^0 \leq G$ (1.2)

22                          $q_L^0 = G_L^0 N_L^0 \leq n_L$ (1.3)

23                          $q_T^0 = G_T^0 N_T^0 \leq n_T$ (1.4)

24                              $G_L^0, G_T^0 \geq 0$ (1.5)

25   Equation (1.1) ensures that the intersection serves the correct balance of both vehicle classes; (1.2)
26 ensures that only the available green time is used; and (1.3) and (1.4) that both the signals and the
27 upstream approach lanes can sustain the flows. The inequalities in (1.3) and (1.4) are redundant if $n_L \geq$
28 GN_L^0 and $n_T \geq GN_T^0$. This is almost always the case.

29   Consideration shows that if n_L and n_T do not enter the picture then the optimum solution of the
30 CLP is:

31                  $$q^0 = \frac{G}{l / N_L^0 + (1-l) / N_T^0}$$ (2.1)

32                              $q_L^0 = q^0 l$ (2.2)

33                          $q_T^0 = q^0(1-l)$ (2.3)

34                          $G_L^0 = q^0 l / N_L^0$ (2.4)

1
$$G_T{}^0 = q^0(1-l) / N_T{}^0 \,.$$
(2.5)

2   Equation (2.1) gives the capacity and (2.4) - (2.5) the signal sub-phase durations. As expected,
3 (1.2) is binding.

4   The maximum flow of the tandem design can also be found with a similar LP, which shall be
5 called "TLP". The main two differences are that now there are four green phase durations to be
6 determined, and that the flows of the two vehicle types are now constrained both by the signal and the
7 pre-signal. If the storage area is long enough to prevent any transient queues at the signal from backing up
8 to the pre-signal their capacities can be evaluated independently. This assumption is made in the LP
9 below:

10   (TLP) max q

11                     $q_L = ql$; $q_T = q(1-l)$ (3.1)

12                          $G_L + G_T \leq G$ (3.2)

13                          $g_L + g_T \leq 1$ (3.3)

14                     $q_L = G_L N_L = g_L n_L$ (3.4)

15                     $q_T = G_T N_T = g_T n_T$ (3.5)

16                     $G_L, G_T, g_L, g_T \geq 0 \,.$ (3.6)

17   New in this program are constraint (3.3) and the last member of constraints (3.4) and (3.5).
18 Although the TLP appears to be more constrained due to these additions, this is not so because the tandem
19 design allows us to allocate more lanes to at least one vehicle type; i.e., $N_L \geq N_L{}^0$, $N_T \geq N_T{}^0$, and $N_L + N_T >$
20 $N_L{}^0 + N_T{}^0$.

21   As before, the reader can verify that the solution of the TLP is:

22
$$q = \min\left\{ \frac{G}{l/N_L + (1-l)/N_T} \,, \frac{1}{l/n_L + (1-l)/n_T} \right\}$$
(4.1)

23                          $q_L = ql$ (4.2)

24                          $q_T = q(1-l)$ (4.3)

25                          $G_L = ql / N_L$ (4.4)

26                          $G_T = q(1-l) / N_T$ (4.5)

27                          $g_L = ql / n_L$ (4.6)

28                          $g_T = q(1-l) / n_T \,.$ (4.7)

29   The second term of (4.1) is the maximum flow that can pass through the pre-signal, and the first
30 term the maximum flow through the signal. For an intersection without turn pockets ($N_L{}^0 = n_L$ and $N_T{}^0 =$
31 n_T), the second term is greater than (2.1) by a factor of $1/G$. Furthermore, since $N_L \geq N_L{}^0$, $N_T \geq N_T{}^0$, and
32 $N_L + N_T > N_L{}^0 + N_T{}^0$, the first term of (4.1) is also greater than (2.1); perhaps considerably so. Thus, for any
33 intersection without turn pockets the tandem system improves capacity.

1   The improvements for typical cases can be quite large. For example, assume that: $G = \frac{1}{2}$; $l = 1/3$;
2    $N_L^0 = 1$; $N_T^0 = 2$; and $n_L = 1$, $n_T = 2$, $N_L = N_T = 3$ as in FIGURE 2a. Then, for the conventional design, (2.1)
3   - (2.3) yield: $q_L^0 = 1/4$; $q_T^0 = 1/2$; and $q^0 = 3/4$; and for the tandem design (4.1) - (4.3) yield: $q_L = 1/2$; $q_T = $
4   1; and $q = 3/2$. Thus, capacity is increased by 100%. The same calculations for the partial design of
5   FIGURE 2b or FIGURE 2c show a capacity increase of 33%. These results are typical. It is however
6   possible to construct examples with turn pockets where the tandem design would reduce capacity. But
7   these examples always involve unreasonably large values of G and/or small values of n / N.

8   *3.1.2   Optimal Design*
9   The above assumes that the lane assignments are given and only the durations of the pre-signal phases
10   and signal sub-phases can be adjusted. In reality lane assignments could be changed in response to the
11   demand. It is therefore of some interest to know the capacity that can be achieved when both the green-
12   phase durations and the lane designations (n_L, n_T, N_L, N_T, N_L^0, and N_T^0) are optimally set. This can be done
13   both for the conventional and tandem configurations by adding the following constraints to the CLP and
14   TLP:

15    $$n_L + n_T = n \text{ for CLP and TLP} \tag{5.1}$$

16    $$N_L^0 + N_T^0 = N \text{ for CLP} \tag{5.2}$$

17    $$N_L, N_T \leq N \text{ for TLP} \tag{5.3}$$

18    $$N_L + N_T - N = k \text{ for TLP with } k \text{ tandem lanes.} \tag{5.4}$$

19   With the new variables, some of the constraints in the CLP and the TLP become non-linear.
20   Furthermore, since the new variables are natural numbers, the resulting problems are mixed-integer non-
21   linear programs. In practical cases the number of feasible lane designations arising from (5) is small.
22   Therefore, these non-linear programs can be easily solved by evaluating (2.1) or (4.1) for all feasible lane
23   designations and choosing those that maximize the result.

24   To identify the proper application domain for the tandem method, a battery of these generalized
25   CLP and TLP problems were solved. Four basic approach configurations were examined: approaches
26   with 2 and 3 lanes, with and without 1-lane turn pockets. Their capacities were evaluated for all possible
27   values of the input parameters (G and l), both under conventional and tandem control. For the sake of
28   brevity, the latter was only analyzed for the case in which only one tandem lane is used; i.e., where its
29   potential benefit is least. FIGURE 3 shows the result. Contour lines show the ratio of the partial tandem
30   capacity over the maximal possible flow (GN). Shading shows the ratio of the partial tandem capacity
31   over the conventional capacity, with lighter shades denoting greater benefits reaped by the tandem design.
32   These figures can also be used to determine if a given pair of flows can be served with a given G. To do
33   so, go to the appropriate figure and determine from the point with the relevant G and l whether the given
34   total flow is larger or smaller than the capacity displayed for the point's contour.

35   Note that when G is very large (approaching 1) the tandem system performs worse than the
36   conventional. This happens because the pre-signal then becomes the binding constraint. But for common
37   settings with G around 0.5, the capacity increase is about 20% to 30%. This is encouraging because the
38   benefit is obtained with only one tandem lane. On the other hand, the results in this sub-section are
39   optimistic because they assume that vehicles discharge from the intersection at a deterministic rate. The
40   next sub-section obtains more realistic results by relaxing this assumption.

1
2   (a)   (b)

3
4   (c)   (d)
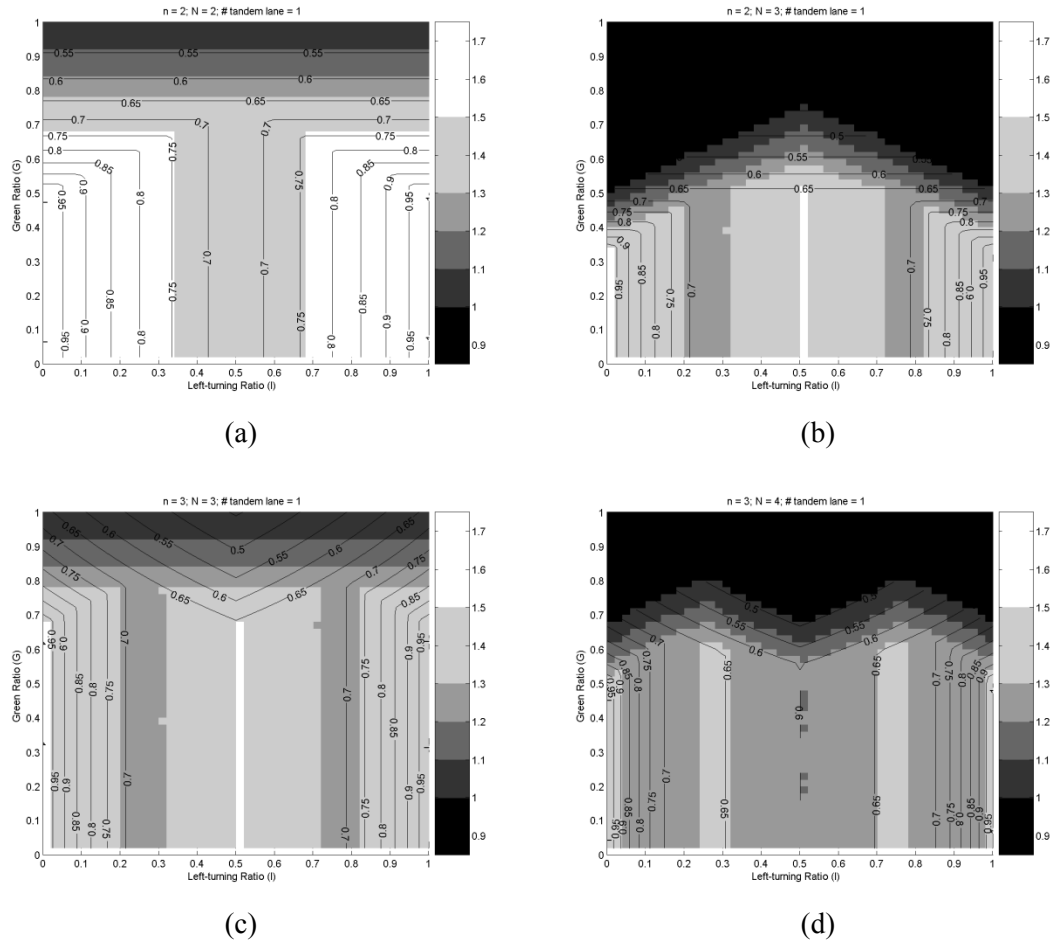
5   **FIGURE 3 Comparison of the deterministic version of capacities of conventional and tandem design. Contour lines show**
6   **the ratio of the tandem capacity over the maximal possible flow; shading shows the ratio of the tandem capacity over the**
7   **conventional capacity: (a) 2 lanes with no turn pocket ($n = N = 2$), 1 tandem lane; (b) 2 lanes with 1-lane turn pocket ($n =$**
8   **$2, N = 3$), 1 tandem lane; (c) 3 lanes with no turn pocket ($n = N = 3$), 1 tandem lane; (d) 3 lanes with 1-lane turn pocket (n**
9   **$= 3, N = 4$), 1 tandem lane.**

10   3.2. Stochastic Considerations

11   The concepts discussed below are easier to understand if the cycle time (C) and the average discharge
12   headway (H) are explicitly used as variables. Therefore, the special system of units we have been using,
13   where these two variables took the value "1", will not be used. Instead the results will be derived for an
14   arbitrary system and then converted to the special system at the end.

15   Refer to FIGURE 2a (full tandem) and consider what would happen if the LVs in the sorting area
16   were unable to fully discharge in their signal sub-phase (call this a sub-phase failure.) The residual LVs
17   would have to stop and wait for the next sub-phase. If they block all the lanes, no TVs would discharge in
18   their sub-phase. Instead, these TVs would have to wait until the residual LVs have cleared the intersection.
19   Since this will happen in the next cycle—ignoring the unlikely event that a batch of LVs may need more
20   than 2 sub-phases to discharge—a full cycle would have been wasted for both vehicle types.

21   It should be clear that if the TV sub-phase fails (and residual TVs block all lanes) then a cycle is
22   still lost for the same reasons; and that if both the TV and TV sub-phases fail back to back, then 2 cycles

1   are lost. Thus, a sub-phase failure of either type, regardless of when it happens relative to other failures,
2   wastes exactly one cycle. This conclusion is slightly pessimistic because some failures can allow some
3   flow of partially blocked vehicles, as is always the case with partial tandem designs. Nonetheless, the
4   result will be used without further refinement in order to produce conservative estimates of potential
5   benefits.

6   To manage failures, we will hold constant the signal sub-phase durations but will reduce the size
7   of the vehicle batches that the pre-signal allows to pass in every cycle. (The pre-signal is assumed to be
8   adaptive, normally releasing batches of fixed size but holding back all vehicles when a sub-phase has
9   failed.) In the analysis that follows we will focus on a single lane and evaluate the probability of the
10   lane's failure for the given batch size. Then, conservatively assuming that all the lanes fail if this lane fails,
11   we shall derive the expected LT and TV flow per cycle for the given batch sizes.

12   To do this, let $m.^s$ be the size of the released batch in one of the lanes of the sorting area, where "."
13   stands for L or T. The expected number of vehicles such a lane can discharge in a sub-phase is
14   $m. = G./H$. If the coefficient of variation of the saturation headways is γ, then the probability of failure
15   $p.$ is:

16
$$p. = \Pr\left(\sum_{i=1}^{m.^s} H_i > m.H \right) = \Phi\left(-\frac{m. - m.^s}{\sqrt{m.^s}\,\gamma} \right),$$

17   where $\Phi(.)$ is the standard normal c.d.f.. This can be written as:

18
$$p. = \Phi(-k.), \text{ where } k. = \frac{m. - m.^s}{\sqrt{m.^s}\,\gamma} \text{ and } m. = G./H . \tag{6}$$

19   The fixed number of vehicles released by the pre-signal in one of its regular cycles (i.e., a cycle
20   without a failure) is: $N_L m_L^s + N_T m_T^s$. The average number of cycles required to serve these vehicles is
21   $(1 + p_T + p_L)$, because every failed sub-phase requires an extra cycle. Thus, the average flow through the
22   signal is: $\left(N_L m_L^s + N_T m_T^s \right) / \left[C\left(1 + p_L + p_T\right) \right]$. This can be rewritten in terms of $k.$ using (6):

23
$$q^s = \frac{N_L G_L / H \left(1 - k_L \gamma / \sqrt{G_L / H}\right) + N_T G_T / H \left(1 - k_T \gamma / \sqrt{G_T / H}\right)}{C\left(1 + \Phi(-k_L) + \Phi(-k_T)\right)} . \tag{7.1}$$

24   To express this equation in the special unit system that was used in Section 3.1 without C or H,
25   replace γ (which is the coefficient of variation of the time it takes to discharge one unit of vehicle quantity)
26   by its counterpart γ' in the special unit system. This counterpart is $\gamma' = \gamma / \sqrt{C/H}$ because in the
27   special unit system a unit of vehicle quantity consists of C/H independent vehicles. Thus the equation
28   becomes:

29
$$q^s = \frac{N_L G_L \left(1 - k_L \gamma' / \sqrt{G_L}\right) + N_T G_T \left(1 - k_T \gamma' / \sqrt{G_T}\right)}{1 + \Phi(-k_L) + \Phi(-k_T)} . \tag{7.2}$$

1   To find the optimum system configuration and signal settings, replace the objective function of
2   TLP with (7.2) and solve the resulting problem. This stochastic version of the design problem will be
3   called STLP. It provides a more realistic, albeit somewhat pessimistic assessment of the system than the
4   TLP. [Note: the STLP does not include extra constraints in connection with the reduced batch sizes, $m_{.}^{s}$,
5   because the deterministic pre-signal green times are always sufficient to accommodate them. To find $m_{.}^{s}$,
6   first obtain $m_{.}$ by inserting the optimal $G_{.}$ in the last equality of (6), and then insert $m_{.}$ and the optimum
7 $k_{.}$ in the second equality of (6).]

8   Note the $k_{.}$ variables only appear in the objective function of the STLP. Thus, to solve it, we first
9   maximize (7.1) or (7.2) with respect to these variables and then solve for the remaining decision variables.
10 Although the first step is analytically challenging, we find numerically from (7.1) that q^s is a very smooth
11 function of k_L and k_T around the optimum. For example, given H = 2.5 s, G/C = 0.5, and γ = 0.2, with C
12 ranging from 30 s to 180 s, and l ranging from 0.1 to 0.9, we find that the value of q^s for $k_L = k_T = 2$
13 always exceeds 99% of the true optimum. Therefore, we fix these variables at $k_L = k_T = 2$ and then solve
14 the STLP for the remaining variables as in section 3.1. This is done by evaluating all feasible
15 combinations of N_L and N_T as in section 3.1.2. The resulting solution should be a conservative
16 approximation of what might be expected in reality. FIGURE 4 shows these solutions for the same battery
17 of problems as in FIGURE 3, using C/H = 48. Note that improvements are still achieved, albeit of a
18 smaller magnitude.

4. DISCUSSION

4.1 Potential benefits

21 Note from Figures 3 and 4 that low green ratios are problematic under stochastic considerations, even
22 though they were fine in the deterministic cases. This happens because the signal processes (C/H) = 48
23 vehicles per lane per cycle times the green ratio; i.e., less than 10 vehicles per lane per cycle if the green
24 ratio is less than 0.2. Such low vehicle numbers magnify the detrimental impact of stochastic fluctuations.
25 However, for more usual green ratios ($0.4 < G < 0.5$), the tandem system with 1 tandem lane increases
26 capacity in all four lane configurations displayed in Figure 4. The benefit is greater if the intersection is
27 narrow and has no turn pockets. In these cases capacity increases by more than 15% when G is between
28 0.3 and 0.7, and l is below 0.2. But the improvement can exceed 30% for certain combinations of G and l.

29   Greater benefits are achieved if one allows for more tandem lanes. This is shown in FIGURE 5,
30 which evaluates the same intersection configurations as before, but with 2 or 3 tandem lanes. As the
31 figure shows, improvements on the order of 50% can be obtained when $G \approx 0.5$ and the turning ratios are
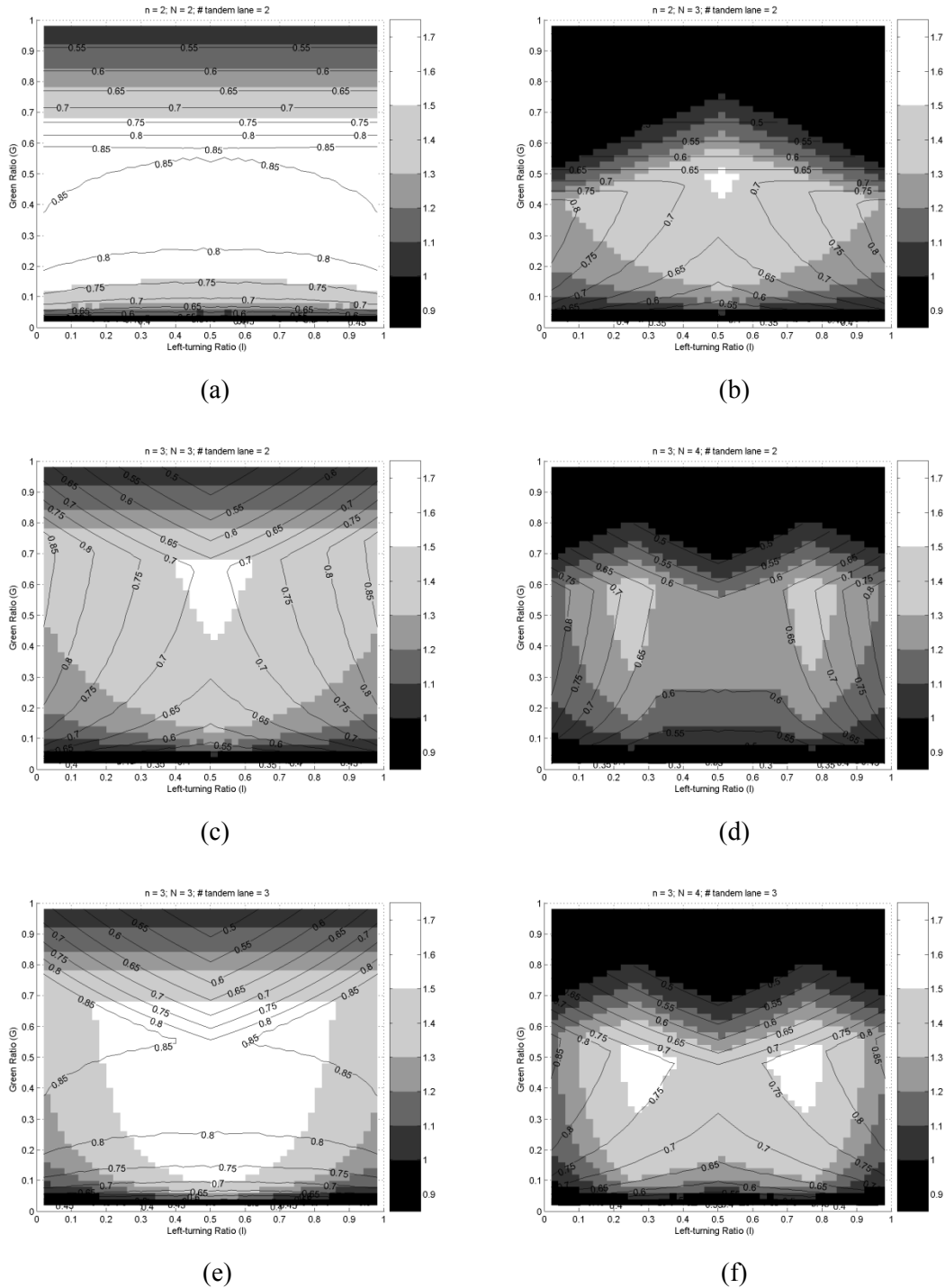32 significant.

1
2   (a)   (b)

3
4   (c)   (d)
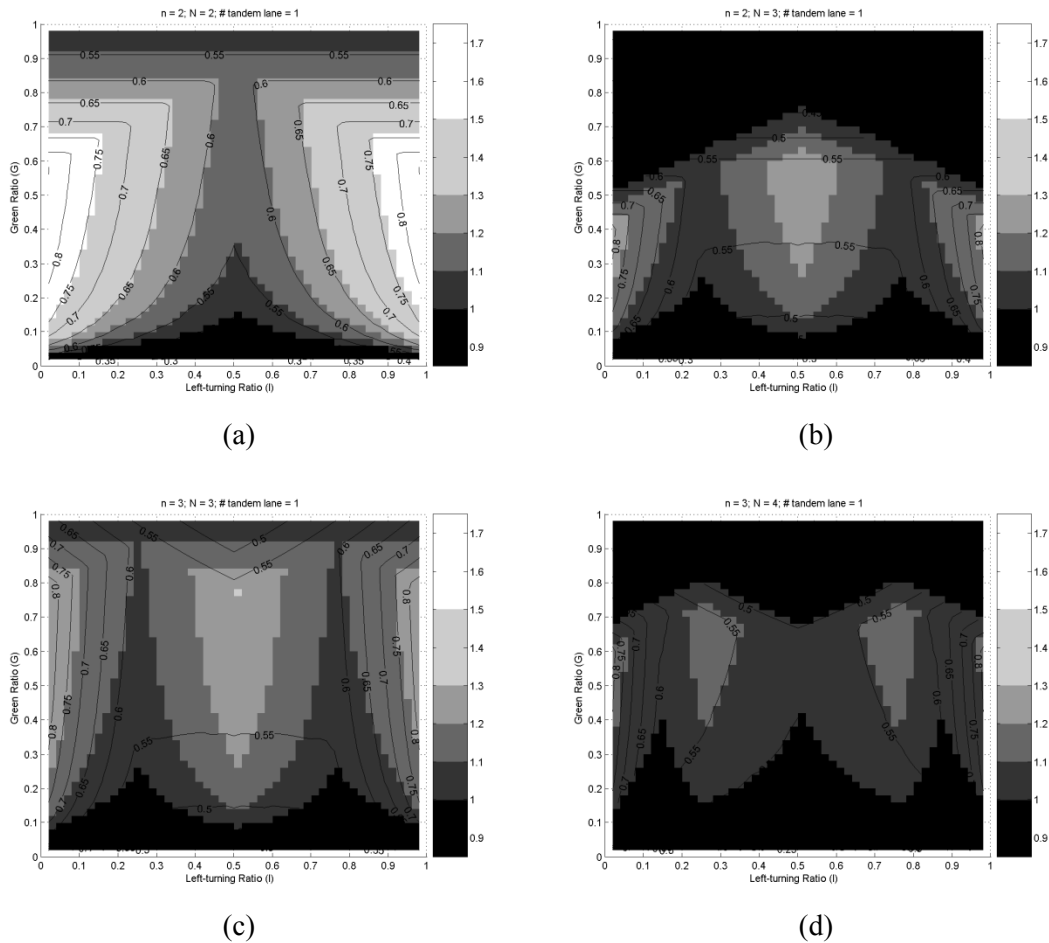
5   **FIGURE 4 Stochastic capacities of conventional and tandem design with *C/H* = 48. Contour lines show the ratio of the**
6   **tandem capacity over the maximal possible flow; shading shows the ratio of the tandem capacity over the conventional**
7   **capacity: (a) 2 lanes with no turn pocket (*n* = *N* = 2), 1 tandem lane; (b) 2 lanes with 1-lane turn pocket (*n* = 2, *N* = 3), 1**
8   **tandem lane; (c) 3 lanes with no turn pocket (*n* = *N* = 3), 1 tandem lane; (d) 3 lanes with 1-lane turn pocket (*n* = 3, *N* = 4),**
9   **1 tandem lane.**

10

11   The reader may complain that the conventional case is unduly penalized in these comparisons
12   because LVs and TVs are not allowed to share lanes in the conventional design—and that a significant
13   penalty would exaggerate the benefits of the tandem configuration. But this penalty turns out to be either
14   non-existent or insignificant. Calculations show that a shared lane actually *reduces* the capacity of the
15   conventional design unless the turning ratio is quite small—and in this case the tandem configuration
16   would not be of much benefit anyway; see TABLE 1. This table shows critical turning ratios, which if
17   exceeded negate the benefit of a shared lane in the conventional configuration. The reason for the
18   ineffectiveness of shared lanes with a conventional design is that during the LV sub-phase, any TV in a
19   shared lane would block the lane from discharging, and vice versa. The tandem strategy in essence
20   removes the blocking; it even removes the limitation to use only one shared lane. This is why the capacity
21   increases in FIGURE 5 are so large.

1
2   (a)   (b)

3
4   (c)   (d)

5
6   (e)   (f)

7   **FIGURE 5 Stochastic capacities of the conventional and tandem designs with *C/H* = 48. Contour lines show the ratio of**
8   **the tandem capacity over the maximal possible flow; shading shows the ratio of the tandem capacity over the conventional**
9   **capacity: (a) 2 lanes with no turn pocket (*n* = *N* = 2), 2 tandem lanes; (b) 2 lanes with 1-lane turn pocket (*n* = 2, *N* = 3), 2**
10   **tandem lanes; (c) 3 lanes with no turn pocket (*n* = *N* = 3), 2 tandem lanes; (d) 3 lanes with 1-lane turn pocket (*n* = 3, *N* = 4),**
11   **2 tandem lanes; (e) 3 lanes with no turn pocket (*n* = *N* = 3), 3 tandem lanes; (f) 3 lanes with 1-lane turn pocket (*n* = 3, *N* =**
12   **4), 3 tandem lanes.**

1   **TABLE 1 Critical left-turning ratios on approaches without turn pockets for a shared lane to be beneficial**

Critical Left-Turning Ratio	n = N = 2			n = N = 3		
	C = 60 s	C = 90 s	C = 120 s	C = 60 s	C = 90 s	C = 120 s
G/C = 0.4	14%	8%	6%	7%	4%	3%
G/C = 0.5	11%	6%	5%	5%	3%	2%
G/C = 0.6	8%	5%	4%	4%	3%	2%

2

4.2   Domain of application

The tandem concept is not intended for intersections that are always undersaturated because in this case extra capacity is not needed; and the concept would actually delay some vehicles a little. It should only be applied to reduce or eliminate oversaturation. Ideally, the tandem system would only be activated during the oversaturated times of day, when it can produce a benefit.

The tandem concept is not feasible with short blocks because it requires extra space to hold the transient queues of the vehicles released into the sorting area. Appendix A of (*15*) derives expressions for the lengths of both, the sorting area and the street section upstream of the pre-signal required to hold its queues without spill-backs. Notice that this total distance is roughly proportional to cycle length—it turns out to be about 100m per minute of cycle length. Since this combined distance cannot exceed the block length, short cycles may be needed if blocks are short. Fortunately, by increasing capacity, pre-signals allow cycles to be shortened. Short cycles can also benefit other users of the intersection, like bicyclists and pedestrians.

Because the tandem concept is new, we do not know how people will react to it. Field tests are being planned to see how it would work in reality.

ACKNOWLEDGMENT

REFERENCES

[1]   Newell, G. F. *Theory of highway traffic signals*. Publication UCB-ITS-RR-89-7, Institute of Transportation Studies, University of California, Berkeley, 1989.

[2]   Webster, F. V. *Traffic signal settings*. Road Research Lab Technical Paper no. 39, 1958.

[3]   Allsop, R. E. Estimating the traffic capacity of a signalized road junction. *Transportation Research*, Vol. 6, No. 3, 1972, pp. 245-255.

[4]   Improta, G., and G. E., Cantarella. Control system design for an individual signalized junction. *Transportation Research Part B*, Vol. 18, No. 2, 1984, pp. 147-167.

[5]   Gallivan, S., and B. G. Heydecker. Optimising the control performance of traffic signals at a single junction. *Transportation Research Part B*, Vol. 22, No. 5, 1988, pp. 357-370.

1   [6]   Silcock, J. P. Designing signal-controlled junctions for group-based operation. *Transportation*
2   *Research Part A*, Vol. 31, No. 2, 1997, pp. 157-173.

3   [7]   Wong, C. K., and S. C. Wong. Lane-based optimization of signal timings for isolated junctions.
4   *Transportation Research Part B*, Vol. 37, No. 1, 2003, pp. 63-84.

5   [8]   Applied Technology and Traffic Analysis Program. *Unconventional arterial intersection design.*
6   University of Maryland, College Park. http://attap.umd.edu/uaid.php. Accessed July 31, 2010.

7   [9]   Rodegerdts, L. A., B. Nevers, and B. Robinson. *Signalized intersections: Informational guide*.
8   Publication FHWA-HRT-04-091. FHWA, U. S. Department of Transportation, 2004.

9   [10] Oakes, J., A. M. ThellMann, and I. T. Kelly. Innovative bus priority measures. *Proceedings of*
10   *Seminar J, Traffic Management and Road Safety, 22nd PTRC European Transport Summer Annual*
11   *Meeting*, Vol. 381, 1994, pp. 301-312.

12 [11] Wu, J., and N. Hounsell. Bus priority using pre-signals. *Transportation Research Part A*, Vol. 32,
13   No. 8, 1998, pp. 563-583.

14 [12] Xuan, Y., V. Gayah, C. F. Daganzo, and M. J. Cassidy. *Multimodal traffic at isolated signalized*
15   *intersections: New manaagement strategies to increase capacity*. Publication UCB-ITS-VWP-2009-
16   10, Volvo Center of Excellence for Future Urban Transport, University of California, Berkeley, 2009.

17 [13] Al-Salman, H. S. T., and R. J. Salter. The control of right turning vehicles at signal controlled
18   intersections. *Traffic Engineering & Control*, Vol. 15 No. 15, 1974, pp. 683-686.

19 [14] Goldblatt, R., F. Mier, and J. Friedman. Continuous flow intersection. *Institute of Transportation*
20   *Engineers Journal*, Vol. 64, No. 7, 1994, pp. 35-42.

21 [15] Xuan, Y., C. F. Daganzo, and M. J. Cassidy. *Increasing the Capacity of Signalized Intersections with*
22   *Separate Left Turn Phases*. Publication UCB-ITS-VWP-2010-6, Volvo Center of Excellence for
23   Future Urban Transport, University of California, Berkeley, 2010.